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Numerical Simulations for 2D Isotropic Solids

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ABSTRACT: Since the isotropic materials have been widely used in the fields of architecture, aviation and so on, the related mechanical calculation has become an important research topic. One of the most commonly used method is the semi inverse method with displacement or stress as the basic variable. However, this method can not describe the local deformation. In this paper, based on the variational principle and the method of separating variables, the basic equations of symplectic system are established. According to the characteristics of Hamiltonian matrix operators, all analytical solutions of transversely isotropic elastic plane are found, and a complete solution space is established. In the example, we derived the first five non-zero eigenvalues.

KEY WORDS: Isotropic, Displacement, Stress.

I. INTRODUCTION

Transverse isotropy is a special case of anisotropy. This kind of material has the same elastic properties in all directions of a plane, but the mechanical properties in all directions perpendicular to the plane are different, so it is called transverse isotropic elasticity. Recently, anisotropic elasticity has been applied to various fields including aviation and architecture due to the needs of engineering design and optimization design, and the related mechanical calculation has also become an important research field. However, compared with isotropic materials, the analytical solution is more difficult.

Knowles and Horgan obtained the decay rate of the expansion problem in an isotropic cylinder by using the stress function [1]. Horgan established H-K method to study the attenuation of end effect in isotropic and transversely isotropic cylinder by using energy attenuation inequality [2]. Ladeveze [3] gave an interpretation of the Saint-Venant principle, in which the localizations of the stresses and displacements are given. However, the above methods are all univariate methods with displacement or stress as basic variables, belonging to the Lagrangian system, which is difficult to avoid the difficulty of solving higher-order partial differential equations [4].

In this paper, a new symplectic approach is applied to study the transversely isotropic boundary condition problems. According to the characteristics of Hamiltonian matrix operators, all the analytical are obtained directly, and the complete eigensolution space is obtained. In the example, the boundary problem with fixed end constraints is studied, in which the local deformation and stress concentration caused by boundary conditions are well described.

II. SOLUTION METHOD

Consider a homogeneous transversely isotropic elastic media of the rectangular domain $(-b \le x \le b,\ -l \le z \le l)$ in the Cartesian coordinate systemas. Applying the principle of minimum potential energy, the Lagrange function without considering the body forces can be expressed as:

$$L(u,w) = \frac{1}{2}\left\{C_{11}\left(\partial_x u\right)^2 + C_{44}\left[\dot{u}^2 + C_{33}\dot{w}^2\right] + \left(\partial_x w\right)^2\right\} + C_{13}\partial_x u\dot{w} + C_{44}\dot{u}\partial_x w \tag{1}$$

where C_{ij} is the elastic stiffness. According to the principle of minimum potential energy, we get the following governing equations in the Hamiltonian system

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$$\dot{\mathbf{q}} = \begin{Bmatrix} C_{44}\dot{u} + C_{44}\dfrac{\partial w}{\partial x} \\ C_{13}\dfrac{\partial u}{\partial x} + C_{33}\dot{w} \end{Bmatrix} \tag{2}$$

and

$$\dot{u} = a_1 p_1 - \frac{\partial w}{\partial x}, \quad \dot{w} = a_3 p_2 - a_2 \frac{\partial u}{\partial x} \tag{3}$$

where

$$a_1 = \frac{1}{C_{44}}, \quad a_2 = \frac{C_{13}}{C_{33}}, \quad a_3 = \frac{1}{C_{33}} \tag{4}$$

The boundary conditions are

$$\begin{aligned} & a_4\frac{\partial u}{\partial x} - a_2 p_2 = \sigma_x \\ & p_1 = \tau_{xz} \end{aligned} \qquad (x = \pm b) \tag{5}$$

For the case of $\mu = 0$, the governing equations are

$$\mathbf{H}\boldsymbol{\psi}_i^{(2)} = \boldsymbol{\psi}_i^{(1)} \qquad (i = 1, 2) \tag{6}$$

The fundamental solutions are

$$\boldsymbol{\eta}_1^{(0)} = \boldsymbol{\psi}_1^{(0)} = \{1 \quad 0 \quad 0 \quad 0\}^T \tag{7}$$

and

$$\boldsymbol{\eta}_2^{(0)} = \boldsymbol{\psi}_2^{(0)} = \{0 \quad 1 \quad 0 \quad 0\}^T \tag{8}$$

It can be proved that the above equations have Jordan type solutions. After theoretical derivation, we get the following four Jordan type solutions

$$\begin{aligned} & \boldsymbol{\psi}_1^{(1)} = \{0 \quad -x \quad 0 \quad 0\}^T \\ & \boldsymbol{\psi}_2^{(1)} = \{a_5 x \quad 0 \quad 0 \quad 1\}^T \end{aligned} \tag{9}$$

$$\boldsymbol{\psi}_1^{(2)} = \left\{-\frac{1}{2}a_5 x^2 \quad 0 \quad 0 \quad -x\right\}^T \tag{10}$$

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and

$$\boldsymbol{\psi}_1^{(3)} = \left\{ 0 \quad a_6 x^3 - \frac{a_1 a^2}{2} x \quad \frac{x^2 - a^2}{2} \quad 0 \right\}^T \tag{11}$$

The final solutions of the problem are

$$\boldsymbol{\eta}_1^{(1)} = \boldsymbol{\psi}_1^{(1)} + z\boldsymbol{\psi}_1^{(0)} = \{ z \quad -x \quad 0 \quad 0 \}^T \tag{12}$$

and

$$\boldsymbol{\eta}_2^{(1)} = \boldsymbol{\psi}_2^{(1)} + z\boldsymbol{\psi}_2^{(0)} = \{ a_5 x \quad z \quad 0 \quad 1 \}^T \tag{13}$$

Using the method of separating variables, we have

$$\boldsymbol{\eta}_1^{(2)} = \boldsymbol{\psi}_1^{(2)} + z\boldsymbol{\psi}_1^{(1)} + \frac{z^2}{2}\boldsymbol{\psi}_1^{(0)} = \left\{ \frac{-a_5 x^2 + z^2}{2} \quad -xz \quad 0 \quad -x \right\}^T \tag{14}$$

The solution is

$$\boldsymbol{\psi} = \boldsymbol{\psi}_1^{(3)} + z\boldsymbol{\psi}_1^{(2)} + \frac{z^2}{2}\boldsymbol{\psi}_1^{(1)} + \frac{z^3}{6}\boldsymbol{\psi}_1^{(0)} \tag{15}$$

Thus

$$\boldsymbol{\eta}_1^{(3)} = \left\{ \frac{z^3}{6} - \frac{a_5 x^2 z}{2} \quad a_6 x^3 - \frac{a_1 a^2 + z^2}{2} x \quad \frac{x^2 - a^2}{2} \quad -xz \right\}^T \tag{16}$$

The boundary conditions are

$$\begin{cases} \int_{-a}^{a} c_2 dx = \int_{-a}^{a} p_{2,-l}^{*} dx \\ \int_{-a}^{a} c_1 \frac{x^2 - a^2}{2} dx = \int_{-a}^{a} p_{1,-l}^{*} dx \\ -\int_{-a}^{a} c_3 x^2 dx + l \int_{-a}^{a} c_1 \frac{x^2 - a^2}{2} dx = \int_{-a}^{a} p_{2,-l}^{*} x dx \end{cases} \tag{17}$$

Based on the property of the Hamiltonian operator matrix $\mathbf{H}$, the integral product of the eigenvectors is defined as

$$< \boldsymbol{\psi}_1, \mathbf{J}, \boldsymbol{\psi}_2 > = \int_{-b}^{b} \boldsymbol{\psi}_1^{\mathrm{T}} \mathbf{J} \cdot \boldsymbol{\psi}_2 dx \tag{18}$$

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An arbitrary solution vector can beexpressed by the combination of these eigenvector

$$\boldsymbol{\psi}_c = \sum_n \left(a_n \boldsymbol{\psi}_n + b_n \boldsymbol{\psi}_{-n} \right) \tag{19}$$

The coefficients are

$$\boldsymbol{\psi}_c = \sum_n \left(a_n \boldsymbol{\psi}_n + b_n \boldsymbol{\psi}_{-n} \right) \tag{20}$$

III. NUMERICAL EXAMPLE

Figs1-3 show the displacement, strain and stress components of the first five non-zero eigenvalues. It can be seen from these figures that the eigen value increases or decreases to a limit position with time, which indicates that the non-zero eigenvalue only has an obvious effect near the end.

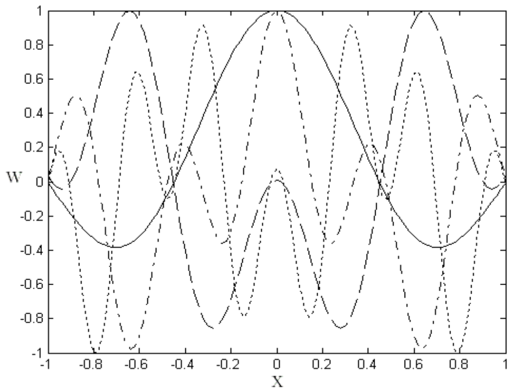

Fig1. Displacement component of the first five non-zero eigenvalues

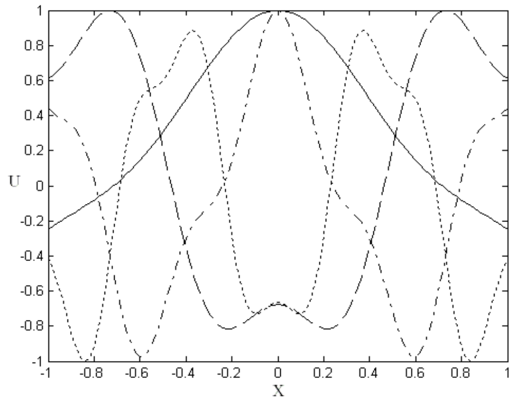

Fig2. Strain component of the first five non-zero eigenvalues

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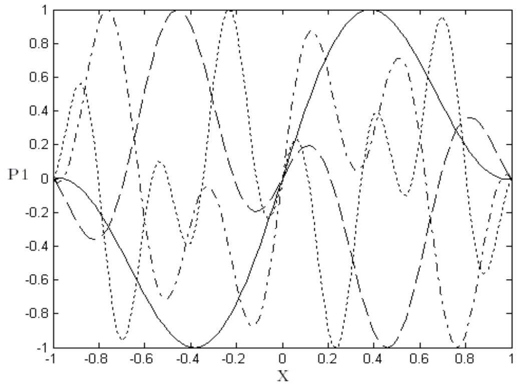

Fig 3.Stress component of the first five non-zero eigenvalues

IV. CONCLUSION AND FUTURE WORK

The Hamiltonian system is applied to find analytical solutions of Saint-Venant problems of elastic cylinders. The study shows that non-zero eigensolutions include the torsion and bending groups charactered by local deformations, while zero eigensolutions are composed of all the overall deformation solutions such as the traditional tension and bending problems. The study shows that zero eigensolutions are composed of all the overall deformation solutions such as the traditional tension and bending problems, while non-zero eigensolutions include the torsion and bending groups charactered by local deformations.

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