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Plane Stationary Problem of the Impact of a Moving Load on a Linearly Compressible Semi-Plane

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ABSTRACT: In this paper, the numerical methods of the characteristics are used to analytically study the effects of the nonlinear properties of a half-plane material on the propagation of shock wave processes in it.A computational scheme is proposed for determining shock wave processes in it. The numerical method of characteristics is compared with the analytical method.

Keywords: The stationary issue is a matter of time. Linear-compressive half-plane-half-plane influenced by moving loads. The wave propagation process is the propagation of the wave shock into the half plane. Strong blast-on-the-moment load on the front, and the rear opposite. Characteristic method is a way of solving he problem.

I. INTRODUCTION

The paper considers a plane stationary problem of the effect of a moving load on a nonlinearly compressible half-plane.

The problem is solved by the numerical method of characteristics, the influence of the nonlinear properties of the half-plane material on the propagation of shock-wave processes in it is analytically studied, and the calculation scheme can be used to determine the parameters of an inhomogeneous medium for various profiles of a given load.

Let a monotonically decreasing normal load move along the surface of the half-plane with a constant speed D exceeding the speed of propagation of load-unloading deformations of the medium. The load profile does not change as the wave propagates.

The medium filling the half-plane possesses such mechanical properties that, under loading and unloading, the relationship between pressure p and volumetric deformation $\boldsymbol{\varepsilon}$ is nonlinear and irreversible, and $\boldsymbol{dp/d\varepsilon} > 0, \boldsymbol{d^2p/d\varepsilon^2} > 0$ and the angle of inclination of the unloading branch of the diagram $p\sim\varepsilon$ exceeds the angle of inclination of the loading branch.

In this case, a shock wave with a curved surface Σ will propagate in the half-plane, the perturbation region is limited by the front Σ and the boundary of the half-plane. It is assumed that the medium at the front Σ is instantly loaded, and unloading occurs behind the front in the disturbed region. On the strong discontinuity surface Σ, from the conditions of conservation of mass and momentum, the following relations hold:

$$\rho_0 a = \rho^*(a - v_n^*), \qquad \rho_0 a v_n^* = \rho^*, \qquad v_\tau^* = 0, \qquad (a = D\sin\alpha). \qquad (1)$$

We represent the equation of state of the medium in the form of a polynomial

$$p^* = \alpha_1\varepsilon^* + \alpha_2\varepsilon^{*2}$$

In the unloading area in a moving coordinate system $\xi = Dt + x, \eta = y$

We have

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$$D\frac{\partial u}{\partial \xi}+\frac{1}{\rho}\frac{\partial p}{\partial \xi}=0, \qquad D\frac{\partial v}{\partial \xi}+\frac{1}{\rho}\frac{\partial p}{\partial \eta}=0, \qquad (2)$$
$$D\frac{\partial p}{\partial \xi}+\rho\left(\frac{\partial u}{\partial \xi}+\frac{\partial v}{\partial \eta}\right)=0,\ \ p=p^*+\beta_1(\varepsilon-\varepsilon^*)+\beta_2(\varepsilon-\varepsilon^*)^2,$$

the boundary condition has the form

at$\eta=0,\quad \xi\ge 0,\quad p=f(\xi)$(3)

where$f(\xi)$ –well-known monotonically decreasing function.

Let's introduce the notation: D – is the speed of the moving load; a -is the speed of propagation of the shock wave;$a_1=c_p=\beta_1/\rho$ the rate of propagation of the unloading deformation for the case $\beta_2=0$; p - pressure;- volumetric deformation; Σ- shock wave front; ρ is the density of the medium;t – time ; x,y – fixed Cartesian coordinates;ξ,η – moving Cartesian coordinates; V is the mass velocity of the medium;u,v –is the projection of the velocity on the ξ and η axes;φ –speed potential;v_n^*,v_τ^*- normal and tangential components of the mass velocity V of the medium to the front Σ; ρ_0 – is the maximum value of the moving load; μ – dimensionless coefficient; b – dimensional factor; α_1, α_2 ,β_1,β_2 –are constant values;α –is the angle of inclination of the front Σ of the shock wave to the boundary of the half-plane;$tg\alpha_0$ –is the tangent of the angle of inclination of the front Σ with the $O\xi$ axis at the origin;the parameters of the medium related to the front Σ are indicated at the top with an asterisk.

II. RESEARCH RESULTS

As mentioned above, the method of characteristics is used to solve the problem, and the main relations on the characteristics in the case of a nonlinearly compressible medium are given in [3].

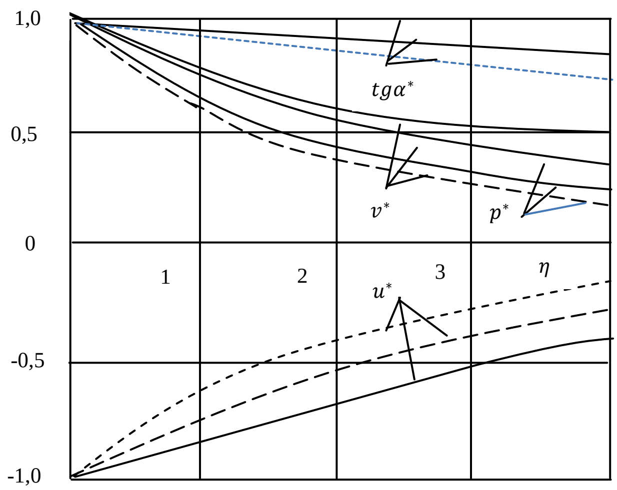

Fig1

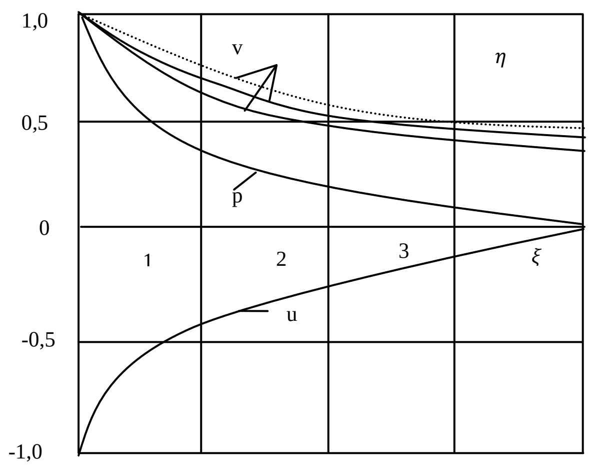

Fig 2

For a specific structure of the medium [4], the problem is implemented on a computer for a load that varies along ξ according to an exponential law of the form

$$f(\xi)=p_0\,exp(-0{,}1\xi)$$

for

$$\alpha_1=12{,}127\cdot 10^2, \alpha_2=58{,}73\cdot 10^3,\quad \beta_1=9{,}016\cdot 10^3,$$
$$\beta_2=19\cdot 10^4,\qquad p^0=\frac{105\text{кг}}{\text{см}^2}$$

and the calculation results are shown in FIG. 1-6, where solid lines refer to the case $\alpha_2\neq\beta_2\neq 0$, dashed lines $-\beta_2=0$, dash-dotted lines $-\alpha_2=\beta_2=0$. The environment parameters in FIG. 1-3 are given in dimensionless form in relation to their maximum value, and the coordinates ξ, η - to the unit of length.

From fig. 1 that the pressure p^* and the velocitiesu^*,v^* vat the front, depending on the depthη , decay in a substantially non-linear manner. It turns out that due to the nonlinearity of the properties of the medium, each material point of the half-plane is in a more stressed state than in the case$\alpha_2=\beta_2=0$. The difference in the parameters

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calculated according to the linear $\alpha_2 = \beta_2 = 0()$ and non linear($\alpha_2 \neq \beta_2 \neq 0$) theory is on average 20-30%, which indicates the need to take into account nonlinear processes occurring in the medium.

Analyzing the dependencies shown in fig. 2, it can be seen that at the boundary of the medium$\eta = 0$ along ξ the velocity componentsu, v monotonically fall (the pressure is given).

The curve of the dependence of the vertical component of the velocity υ on ξ at $\alpha_2 = \beta_2 = 0$ lies above the curve relating to the case $\beta_2 = 0$, and the curve calculated for the nonlinear case lies below this curve. At ξ> 30, when the pressure becomes insignificant, the curves for v at $\beta_2 = 0$ and $\alpha_2 = \beta_2 = 0$coincide. The curves for u everywhere on the boundary of the half-plane are obtained in all cases the same with an accuracy of the line thickness. Curves 1–3 in Fig. 3 show changes in the parameters of the medium in the sections ξ = 0.5; 1; 3, respectively, depending on the depth of propagation of the wave, which asymptotically reaches its maximum (at the front) value, represented for the above two cases by curves 1. Hence, we note that in the case of nonlinear unloading, in comparison with the linear one, the values of p and u are slightly larger, and the curves for υ have intersection points.

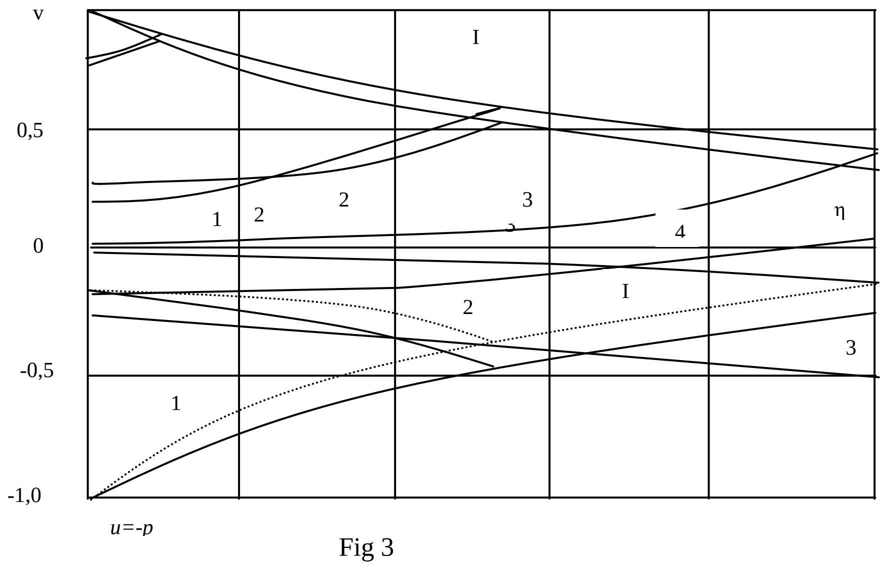

Fig 3

The change in the tangent of the angle of inclination of the shock front to the boundary of the medium is shown in fig. 4, where it is noted that the nonlinear properties of the material of the medium lead to curvature of the wave front and the front velocity decays along the depth of the half-plane. Moreover, the largest angle of inclination at a fixed ξ corresponds to the case of nonlinear loading and unloading of the medium. The curve corresponding to the case of only nonlinear loading lies below the curve of nonlinear loading and unloading, the straight line refers to the linear theory.

Consequently, the nonlinear dependence between the parameters of the medium p and ε leads to an expansion of the perturbation region.

The relationship between the maximum value of the vertical component of the mass velocity $v_{max}$and $tg\alpha$ of the front is shown in fig. 5, which confirms that if the wave front tends to the boundary of the medium, then v_{max}increases.

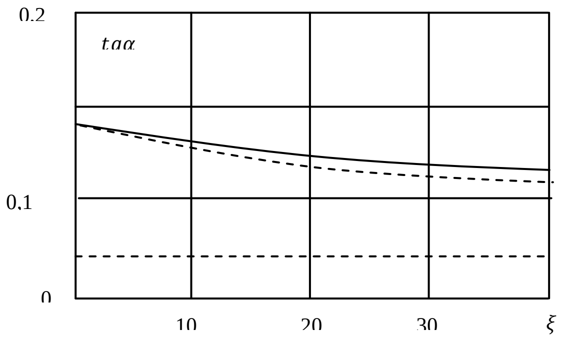

Fig 4

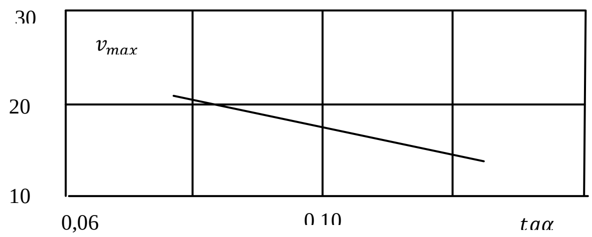

Fig 5

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The dashed line with circles shown in fig. 6 corresponds to the pressure distribution p^* along the front Σ at $\beta_2 = 0$ for the case of approximation of the load branch of the p ~ ε diagram by a chord passing through the points $p = 0$ and $p = p_0$. This pressure curve as a function of η is located above the pressure curve under nonlinear loading.

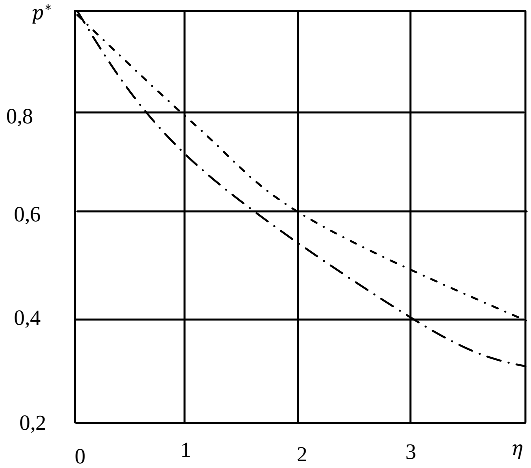

Fig 6

Thus, in the study by the method of characteristics of the influence of nonlinear dependences between the parameters of the medium on the propagation of stress waves in it, it was shown that the nonlinear dependence between p and ε leads to an expansion of the perturbation region, an increase in pressure and velocity in comparison with linear theory. In this case, the parameters *p, u, v,* as well as the propagation velocity of the shock wave in the medium under consideration, become monotonically decreasing functions of the half-space depth.

The study of the system of equations (2) shows that for $\beta_2 = 0$ the problem can be solved analytically. Indeed, substituting the first equation in (2) into the third, for the velocity potential φ we obtain the wave equation

$$\mu^2 \frac{\partial^2 \varphi}{\partial \xi^2} - \frac{\partial^2 \varphi}{\partial \eta^2} = 0 \qquad \left(\mu^2 = \frac{D^2}{c_p^2} - 1, c_p^2 = \frac{\beta_1}{p}\right), \qquad (4)$$

which for $D > c_p$ has a solution of the form

$$\varphi(\xi, \eta) = f_1(\xi - \mu\eta) + f_2(\xi + \mu\eta) \qquad (5)$$

Here, the unknown functions f_1 and f_2 must be determined from the boundary condition (3) and the condition at the wave front (1). Below we propose to solve this problem by the inverse method, i.e. given a certain shape of the wave front surface Σ and in the process of solving the problem, determine the corresponding load profile. In this case, inside the curvilinear sector *ξ*0Σ for (4) we obtain the Cauchy problem, since if the front surface Σ is given (in this case, the front velocity is considered a function that decreases with the depth of the half-plane), then, taking into account (1), all parameters, including the components the velocities of the medium *u, v* on it will be known variable quantities and, for *η = η (ξ)*, have the form

$$u = \frac{\partial \varphi}{\partial \xi} = -D \sin^2 \alpha(\xi) \left[\frac{\rho_0 D^2}{\alpha_2} sin^2 \alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right],$$

$$v = \frac{\partial \varphi}{\partial \eta} = D \sin \alpha\,(\xi) cos\alpha(\xi) \left[\frac{p_0 D^2}{\alpha_2} sin^2 \alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right], \qquad (6)$$

where *η (ξ)* is the equation of the front surface. Using (6), from (5) we find

$$f_i(z_i) = \mp \frac{D}{2\mu} \int_0^{z_i} \frac{\mathrm{tg}\,\alpha[F_i(z_i)]\,\{1 \pm \mu tg\alpha[F_i(z_i)]\}\Phi_i(z_i)}{\{1 + tg^2\alpha[F_i(z_i)]\}^2} dz_i, \qquad (7)$$

where $\Phi_i(z_i) = (p_0 D^2/\alpha^2 - \alpha_1/\alpha_2)\,\mathrm{tg}^2\,\alpha\,[F_i(z_i)] - \alpha_1/\alpha_2\,; F_i(z_i)$is the root of the equation $\xi \pm \mu\eta(\xi) = z_i$, and in the case $i = 1$the upper sign in formula (7) is taken. Thus, in the domain
*ξ*0Σ, taking into account (7), (5), an analytical solution to the problem is obtained. If we substitute this solution in (3), then, in principle, a monotonically decreasing load profile with a sharp front should be obtained at the origin and in the disturbed region, the process of unloading the medium should be carried out. The calculation results show that the process of unloading in the sector ξ0Σ can be achieved if the speed of the wave front Σ is a decaying function over the half-plane depth (which was required to be proved). A similar inverse method was applied to the problem of an unloading wave [5].

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As an illustration of the method, the case is considered when the surface Σ of the wave front is given as a polynomial of the second degree

$$\eta(\xi) = \text{tg}\,\alpha_0 \cdot \xi - (b/2)\xi^2. \quad (8)$$

Table 1

ξ	u^*		v^*		p^*	
	I	*II*	*I*	*II*	*I*	*II*
0	*-1,644*	*-1,644*	*13,100*	*13,100*	*105*	*105*
0,2	*-1,635*	*-1,633*	*13,040*	*13,02*	*104,412*	*104,2*
0,4	*-1,622*	*-1,621*	*12,953*	*12,95*	*103,542*	*103,5*
0,6	*-1,610*	*-1,610*	*12,876*	*12,87*	*102,787*	*102,8*
0,8	*-1,598*	*-1,598*	*12,800*	*12,80*	*102,038*	*102,0*
1,0	*-1,587*	*-1,587*	*12,725*	*12,73*	*101,293*	*101,3*

Note. I – numerical method of characteristics, II – analytical method.

Table 2

ξ	u		v		p	
	I	*II*	*I*	*II*	*I*	*II*
0	*-1,644*	*-1,644*	*13,100*	*13,100*	*105*	*105*
0,2	*-1,610*	*-1,613*	*12,944*	*12,94*	*102,921*	*102,979*
0,4	*-1,581*	*-1,581*	*12,780*	*12,78*	*100,882*	*100,937*
0,6	*-1,550*	*-1,551*	*12,621*	*12,62*	*98,888*	*99,021*
0,8	*-1,519*	*-1,520*	*12,466*	*12,47*	*96,928*	*97,042*
1,0	*-1,490*	*-1,490*	*12,314*	*12,32*	*95,009*	*95,127*

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Note. I – numerical method of characteristics, II – analytical method.

III. CONCLUSION

The results of calculations of the analytical method taking into account (8) at $\mathrm{tg}\,\alpha_0 = 0.1255, b = 0{,}86 \cdot 10^{-3}$ and the method of characteristics described above are presented in table. 1, 2, whence it is seen that the results obtained using both methods are in mutually satisfactory agreement, and the load profile f (ξ) found by the inverse method is monotonically decreasing along ξ.

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