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AGM, AHM, GHM & AGHM: Evaluation of Parameter μ of the Model $X = \mu + \varepsilon$

Dhritikesh Chakrabarty

Associate Professor, Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India

ABSTRACT: Recently four formulations of average have been derived from the Pythagorean means namely Arithmetic Mean (AM), Geometric Mean (GM) and Harmonic Mean (HM). These four formulations namely Arithmetic-Geometric Mean (abbreviated as *AGM*), Arithmetic-Harmonic Mean (abbreviated as *AHM*), Geometric-Harmonic Mean (abbreviated as *GHM*) and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) respectively. Each of these four has been shown to be improved measure of the value of parameter μ of the model $X = \mu + \varepsilon$. This paper is a description on how to evaluate the value of the parameter μ with numerical example / application.

KEYWORDS: AGM, AHM, GHM, AGHM, Central tendency of data, Measure.

I. INTRODUCTION

Several research had already been done on developing definitions / formulations of average [1 , 2], a basic concept used in developing most of the measures used in analysis of data. Pythagoras [3], the pioneer of researchers in this area, constructed three definitions / formulations of average namely Arithmetic Mean, Geometric Mean & Harmonic Mean which are called Pythagorean means [4 , 5 , 14 , 18]. A lot of definitions / formulations have already been developed among which some are arithmetic mean. geometric mean, harmonic mean, quadratic mean, cubic mean, square root mean, cube root mean, general *p* mean and many others [6 , 7 , 8 , 9 , 10 , 11 , 12 , 13 , 14 , 15 , 16 , 17, 18 , 19]. Kolmogorov [20] formulated one generalized definition of average namely Generalized *f* - Mean. [7 , 8]. It has been shown that the definitions/formulations of the existing means and also of some new means can be derived from this Generalized *f* - Mean [9 , 10]. In an study, Chakrabarty formulated one generalized definition of average namely Generalized f_H – Mean [11]. In another study, Chakrabarty formulated another generalized definition of average namely Generalized f_G – Mean [12 , 13] and developed one general method of defining average [15, 16 , 17] as well as the different formulations of average from the first principles [19].
In many real situations, observed numerical data

$$x_1 , x_2 , \ldots\ldots , x_n$$

are found to be composed of a single parameter μ and corresponding chance / random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

i.e. the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots , N \qquad (1.1)$$

[21 , 22 , 23 , 24 , 25 , 26 , 27, 28 , 29].
The existing methods of estimation of the parameter μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [31 – 52] cannot provide appropriate value of the parameter μ [21 , 22 , 23]. In some recent studies, some methods have been developed for determining the value of parameter from observed data containing the parameter itself and random error [21 ,22 , 23 , 24 , 25 , 26, 27 , 28 , 29 , 30 , 53 ,54 , 55 , 56, 57 , 58 , 59 , 60]. The methods, developed in this studies, for determining the appropriate value of the parameter from observed data containing the parameter itself and random error involve huge computational tasks. Moreover, a finite set of observed data may not yield the appropriate value of the parameter in many situations while the number of observations required in the methods may be too large for obtaining the appropriate value of the parameter. However, the appropriate value of the parameter is not perfectly attainable in practical situation. What one can expect is to obtain that value which is more and more close to the appropriate value of the parameter. In order to obtain such value of parameter, four methods have already been developed which involves lesser computational tasks than those involved in the earlier methods as well as which can be applicable in the case of finite set of data [61 , 62 , 63 , 64]. The methods developed are based on the concepts of Arithmetic-Geometric Mean

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(abbreviated as *AGM*) [61 , 62, 66 , 67], Arithmetic-Harmonic Mean (abbreviated as *AHM*) [63], Geometric-Harmonic Mean (abbreviated as *GHM*) [64] and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) [65] respectively.These four formulations namely Arithmetic-Geometric Mean (abbreviated as *AGM*), Arithmetic-Harmonic Mean (abbreviated as *AHM*), Geometric-Harmonic Mean (abbreviated as *GHM*) and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) respectively. Each of these four has been shown to be improved measure of the value of parameter μ of the model $X = \mu + \varepsilon$. This paper is a description on how to evaluate the value of the parameter μ with numerical example / application.

II. AGM, AHM, GHM & AGHM

Let a_0 , g_0 & h_0 be respectively the Arithmetic Mean (*AM*), the Geometric Mean (*GM*) & the Harmonic Mean (*HM*) of N positive numbers or values or observations (not all equal or identical)

$$x_1 , x_2 , \ldots\ldots , x_N$$

all of which are not equal

Then, by the inequality of Pythagorean means [4 , 5] namely

$$AM > GM > HM$$

it follows that

$$h_0 < g_0 < a_0$$

provided $x_1 , x_2 , \ldots\ldots , x_N$ are not all equal.

ARITHMETIC-GEOMETRIC (AGM)

Let us define the two sequences $\{a_n\}$ & $\{g_n\}$ respectively defined by

$$a_{n+1} = ½\, a_n + g_n) ,$$
$$\& \quad g_{n+1} = (a_n\, g_n)^{1/2}$$

& the square root takes the principal value.

Then, the two sequences $\{a_n\}$ & $\{g_n\}$ converge to a common point M_{AG} as n approaches infinity.

This common converging point M_{AG} can be termed / named / regarded as the Arithmetic-Geometric Mean (abbreviated as *AGM*) of the *N* numbers (or values or observations) $x_1 , x_2 , \ldots\ldots , x_N$ [61 , 62, 66 , 67].

ARITHMETIC-HARMONIC MEAN (AHM)

Let $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{h'_n = h'_n(a_0 , h_0)\}$ be two sequences defined by

$$a'_{n+1} = ½(a'_n + h'_n)$$
$$\& \quad h'_{n+1} = ½(a'^{-1}_n + h'^{-1}_n)\}^{-1}$$

respectively.

Then, the two sequences $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{h'_n = h'_n(a_0 , h_0)\}$ converge to common point M_{AH} as n approaches infinity.

This common converging point M_{AH} can be termed / named / regarded as the Arithmetic-Harmonic Mean (abbreviated as *AHM*) of the *N* numbers (or values or observations) $x_1 , x_2 , \ldots\ldots , x_N$ [63].

GEOMETRIC-HARMONIC MEAN (GHM)

Let $\{g''_n\}$ & $\{h''_n\}$ be two sequences defined respectively by

$$g''_{n+1} = (g''_n \,.\, h''_n)^{1/2}$$
$$\& \quad h''_{n+1} = \{½(g''^{-1}_n + h''^{-1}_n)\}^{-1}$$

where the square root takes the principal value.

Then, the two sequences $\{g''_n\}$ & $\{h''_n\}$ converge to common point M_{GH} as n approaches infinity.

This common converging point M_{GH} can be termed / named / regarded as the Geometric-Harmonic Mean (abbreviated as *GHM*) of the *N* numbers (or values or observations) $x_1 , x_2 , \ldots\ldots , x_N$ [64].

ARITHMETIC-GEOMETRIC-HARMONIC MEAN (AGHM)

The three sequences $\{\mathbf{a}'''_n\}$, $\{\mathbf{g}'''_n\}$ & $\{h'''_n\}$ defined respectively by

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$$a'''_n = 1/3\,(a'''_{n-1} + g'''_{n-1} + h'''_{n-1}) \;,$$
$$\mathbf{g}'''_n = (a'''_{n-1}\, g'''_{n-1}\, h'''_{n-1})^{1/3}$$
$$\&\;\; h'''_n = \{1/3\,(a'''^{\,-1}_{n-1} + g'''^{\,-1}_{n-1} + h'''^{\,-1}_{n-1})\}^{-1} \qquad (2.3)$$

converges to a common limit M_{AGH} as n approaches infinity.
This common converging point M_{AGH} can be termed / named / regarded as the Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) of the *N* numbers (or values or observations) $x_1 , x_2 , \ldots\ldots\ldots , x_N$ [65].

III. AGM, AHM, GHM & AGHM AS TOOLS OF EVALUATION OF μ

If the observations

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

are composed of some parameter μ and random errors then the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , N)$$

where

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_N$$

are the random errors associated to

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

respectively which assume positive real values and negative real values in random order.
The parameter μ, in this case, can be interpreted as the central tendency of the observations

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

Let

$$a_0 = AM\,(x_1 , x_2 , \ldots\ldots\ldots , x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i \;,$$
$$g_0 = GM\,(x_1 , x_2 , \ldots\ldots\ldots , x_N) = \left(\prod_{i=1}^{N} x_i \right)^{1/N}$$
$$\&\;\; h_0 = HM\,(x_1 , x_2 , \ldots\ldots\ldots , x_N) = \left(\frac{1}{N} \sum_{i=1}^{N} x_i^{-1}\right)^{-1}$$

Then, the following four results can be obtained:

(1) The two sequences $\{a_n\}$ & $\{g_n\}$ respectively defined by

$$a_{n+1} = ½\, a_n + g_n) \;,$$
$$\&\;\; g_{n+1} = (a_n\, g_n)^{1/2}$$

converge to a common point which is very close to μ [61 , 62].

(2) The two sequences $\{a'_n\}$ & $\{h'_n\}$ respectively defined by

$$a'_{n+1} = ½(a'_n + h'_n)$$
$$\&\;\; h'_{n+1} = ½(a'^{\,-1}_n + h'^{\,-1}_n)\}^{-1}$$

converge to a common point which is very close to μ [63]

(3) The two sequences defined respectively by

$$g''_{n+1} = (g''_n \,.\, h''_n)^{1/2}$$
$$\&\;\; h''_{n+1} = \{½(g''^{\,-1}_n + h''^{\,-1}_n)\}^{-1}$$

converge to a point which is very close to μ [64].

(4) The three sequences $\{\mathbf{a}'''_n\}$, $\{\mathbf{g}'''_n\}$ & $\{h'''_n\}$ defined respectively by

$$a'''_n = 1/3\,(a'''_{n-1} + g'''_{n-1} + h'''_{n-1}) \;,$$
$$\mathbf{g}'''_n = (a'''_{n-1}\, g'''_{n-1}\, h'''_{n-1})^{1/3}$$
$$\&\;\; h'''_n = \{1/3\,(a'''^{\,-1}_{n-1} + g'''^{\,-1}_{n-1} + h'''^{\,-1}_{n-1})\}^{-1}$$

converge to a common point which is very close to μ [65].

Thus, *AGM, AHM, GHM* & *AGHM* of $x_1 , x_2 , \ldots\ldots\ldots , x_N$ converge to some points (may or may not be identical) which are very close to μ.
This implies that the common value of *AGM, AHM, GHM* & *AGHM* of $x_1 , x_2 , \ldots\ldots\ldots , x_N$, if exists, is the value of the parameter μ.
However, if the common value of *AGM, AHM, GHM* & *AGHM* of $x_1 , x_2 , \ldots\ldots\ldots , x_N$ does not exists (or is not found) up to desired decimal place then *AGM, AHM, GHM* & *AGHM* of

$$AGM\,(x_1 , x_2 , \ldots\ldots\ldots , x_N) \;,$$

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$$AHM\,(x_1\,,x_2\,,\;\ldots\ldots\ldots\,,x_N)\,,$$
$$GHM\,(x_1\,,x_2\,,\;\ldots\ldots\ldots\,,x_N)$$
$$\&\;AGHM\,(x_1\,,x_2\,,\;\ldots\ldots\ldots\,,x_N)$$

already obtained can be computed in order to obtain improved value of μ.
The process is required to be repeated if necessary.

IV. NUMERICAL EXAMPLE: APPLICATION TO NUMERICAL DATA

Observed data considered here are the data on each of annual maximum & annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013. The objective here is to evaluate the central tendency of each of annual maximum & annual minimum of surface air temperature at Guwahati

A. Central Tendency of Annual Maximum of Surface Air Temperature at Guwahati

From the observed data on annual maximum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, the values (in Degree Celsius) of *AM*, *GM* & *HM* have been found as follows:

$$AM = 37.2093023255814\ ,$$
$$GM = 37.1922871485760$$
$$\&\ HM = 37.17539890356262$$

[61 ,62 , 63 , 64].
From these, the following results have been obtained:

$$AGM = 37.20079425067069371656824015813\ ,$$
$$AHM = 37.18811147922228321843829512744 9\ ,$$
$$GHM = 37.18384158788008150488383097978 6$$
$$\&\ AGHM = 37.19232688378569045281501129744 1$$
$$(\ 37.183841587880081504883830979786\ \ ,\ \ 37.20079425067069371656824015813\ )$$

Now, the common converging point / value of *AGM*, *AHM*, *GHM* & *AGHM* is 37.
Hence, 37.18 Degree Celsius can be treated as the value of central tendency of Annual Maximum of Surface Air Temperature at Guwahati.

B. Central Tendency of Annual Minimum of Surface Air Temperature at Guwahati

From the observed data on annual maximum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, the values (in Degree Celsius) of *AM*, *GM* & *HM* have been found as follows:

$$AM = 7.36341463414634146341463414634 15\ ,$$
$$GM = 7.25971761945761856087096163512 97$$
$$\&\ HM = 7.15439338028235252098497447075 69$$

[61 , 62 , 63 , 64].
From these, the following results have been obtained:

$$AGM = 7.31147420703016646412362218358 25\ ,$$
$$AHM = 7.25815161833994661021742795089 2\ ,$$
$$GHM = 7.20676689513737000737937277008 02\ ,$$
$$\&\ AGHM = 7.25867355712886575553931587745 38$$

Now, the common converging point / value of *AGM*, *AHM*, *GHM* & *AGHM* is 7.
Hence, 7 Degree Celsius can be treated as the value of central tendency of Annual Minimum of Surface Air Temperature at Guwahati.

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V. RESULTS AND DISCUSSIONS

In the methods developed earlier, for determining the value of parameter from observed data containing the parameter itself and random error, a finite set of observed data may not be sufficient for obtaining the value of the parameter. However, the applications of *AGM*, *AHM*, *GHM* & *AGHM* [61, 62, 63, 64 , 65] can yield the value of the parameter even if the set of observed data is small. Similarly, the application of *AGHM* can also yield the value of the parameter even if the set of observed data is small.

It seems that the application of *AGM*, *AHM*, *GHM* & *AGHM* can yield the value of parameter in the situation under study which is very to the actual value of the parameter.

It has been found that the common converging point / value of *AGM*, *AHM*, *GHM* & *AGHM* in case of Annual Maximum of Surface Air Temperature at Guwahati is 37.
Hence, 37 Degree Celsius can be treated as the value of central tendency of Annual Maximum of Surface Air Temperature at Guwahati which is correct up to two numeric places.
However, from the value of *AGM*, *AHM*, *GHM* & *AGHM*, it is found that as the value of central tendency of Annual Maximum of Surface Air Temperature at Guwahati lies in the interval

(37.183841587880081504883830979786 Degree Celsius , 37.20079425067069371656824015813 Degree Celsius)

Similarly, 37 Degree Celsius can be treated as the value of central tendency of Annual Minimum of Surface Air Temperature at Guwahati which is correct up to one numeric places.
However, from the value of *AGM*, *AHM*, *GHM* & *AGHM*, it is found that as the value of central tendency of Annual Minimum of Surface Air Temperature at Guwahati lies in the interval

(7.2067668951373700073793727700802 Degree Celsius , 7.3114742070301664641236221835825 Degree Celsius)

Further, it can be concluded that the four formulations of average viz.

1. Arithmetic-Geometric Mean (AM).
2. Arithmetic-Harmonic Mean (AHM).
3. Geometric-Harmonic Mean (GHM).
4. Arithmetic-Geometric-Harmonic Mean (AGHM).

can be more accurate measures of central tendency of data than AM , GM & HM.

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AUTHOR'S BIOGRAPHY

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014

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securing 1st class and Senior Diploma (in Guitar) from Prayag Sangeet Samiti in 2019 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty is also an awardee of the Post Doctoral Research Award by the University Grants Commission for the period 2002─05.

Dr. Dhritikesh Chakrabarty is also an awardee of the Post Doctoral Research Award by the University Grants Commission for the period 2002─05.

He attended five of orientation/refresher course held in Gauhati University, Indian Statistical Institute, University of Calicut and Cochin University of Science & Technology sponsored/organized by University Grants Commission/Indian Academy of Science. He also attended/participated eleven workshops/training programmes of different fields at various institutes.

(Dr. Dhritikesh Chakrabarty with students and one employee in Tukurapara Primary School in Gunotsav, 2018)

Dr. Dhritikesh Chakrabarty joined the Department of Statistics of Handique Girls' College, Gauhati University, as a Lecturer on December 09, 1987 and has been serving the institution continuously since then. Currently he is in the position of Associate Professor (& Ex Head) of the same Department of the same College. He had also been serving the National Institute of Pharmaceutical Education & Research (NIPER), Guwahati, as a Guest Faculty continuously from

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May, 2010 to December,2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years. Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 234 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002─05) and one minor research project (2010─11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Referee of the Journal of Assam Science Society (JASS) and a Member of the Editorial Boards of the two Journals namely (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET) and (2) Journal of Mathematics and System Science. Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.