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Comparative Analysis of Composite Barker Codes and Composite Walsh Functions in Data Transmission Systems

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ABSTRACT: The article discusses the correlation properties of pseudo-random sequences (PRS) used in the formation of noise-like signals in data transmission systems and systems with code division multiple access. The comparative analysis of correlation properties of the proposed PRSs, carried out by simulation in Matlab, has shown that the autocorrelation functions of composite Barker codes have better autocorrelation properties, but the composite Walsh functions have better cross-correlation characteristics. The possibility of using these signals in the development of systems with code division channels in order to reduce the level of interference of multiple access is justified.

KEYWORDS: Pseudo-random sequences, noise-like signals, autocorrelation function, cross correlation function, direct composite Barker code, inverse composite Barker code, direct composite Walsh function, inverse composite Walsh function

I. INTRODUCTION

Sufficient experience in the use of broadband communication systems (BBCS) has confirmed their advantages, such as high resistance to narrowband interference, the ability to operate multiple subscribers in one communication channel, transmission secrecy, high resistance to multipath propagation.

For data transmission systems with code division multiplexing, in comparison with other types of systems, it is possible to reuse (multiple) the frequency resource due to the division of channels not by frequency or by time, but by "form", which allows the simultaneous operation of many subscribers in one and the same frequency band.

Such a system uses pseudo-random sequences with specified correlation properties. And the channel signals themselves, formed by expanding the information signal with pseudo-random sequences, are called noise-like; for any other receiver that knows nothing about the spreading sequence, such a signal is noise [1-4]. Pseudo-random sequences are widely used to generate noise-like signals (NLS) in communication systems with direct-sequence spread spectrum (DSSS) or frequency hopping spread spectrum (FHSS). Examples of such systems are DS-CDMA, GPS / Navstar, Glonass, and IEEE 802.11b wireless networks.

The dominant importance in the choice of PRS type for the NLS formation in data transmission systems have, first of all, mutual and autocorrelation characteristics of signals ensemble, as well as its volume, the implementation simplicity of for signals formation and "compression" (convolution) devices in the receiver [1,2,3].

II. THEORETICAL PART

The correlation functions of complex noise-like signals are determined by the correlation functions (CF) of the manipulating sequences. Therefore, considering the CF of complex signals, it is sufficient to analyze the correlation functions of the manipulating sequences. Signal decoding on the receiving side is carried out by a correlation receiver, based on a correlator, which is a series-connected multiplier and an integrator that calculates the cross-correlation function of the incoming signal with the stored in the memoryPRS.

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In case of coincidence of the received sequence and the PRS stored in the memory, the receiver switches to the receiving mode and starts the operation of useful information decoding. In the absence of noise or interference from other signal sources, the output of the correlator will be a signal proportional to the number of matches of the chips of the received and stored code sequence, minus the number of mismatches. Any partial matchescan lead to false triggering and disruption of the receiver.

Consequently, the most important parameter of the used pseudo-random sequences is their correlation properties. Moreover, from the choice of binary code sequences, i.e.from their correlation properties the noise immunity of the entire information system depend on. In addition, the code sequence must be well balanced, that is, the number of ones and zeros in it must differ by no more than one symbol. The last requirement is important to exclude the constant component of the information signal.

The cross-correlation function of two signals $u(t)$ and $v(t)$ is equal to the scalar product $u(t)$ with a copy of $v(t)$ shifted by $t_o$as a function of the argument t_o:

$$R_{uv}(t_o) = \int_{-\infty}^{\infty} u(t)v(t - t_o)dt. \qquad (1)$$

However, based on the generalized Rayleigh formula, we can write:

$$R_{uv}(t_o) = \frac{1}{2pi}\int_{-\infty}^{\infty} U(w)V^*(w)\exp(jwt_o)dw. \quad (2)$$

In asynchronous systems, orthogonality of signals is required regardless of the shift t_o; therefore, for any t_o, the CCF should be zero. However, due to the linearity of the Fourier transform, this is possible only if $U(w)\cdot V^*(w) = 0$ on the entire frequency axis. Equality of CCF to zero means that two signals are orthogonal for any $t_o$only if their spectra do not overlap. However, this cannot be achieved in a multiple access system. The consequence of this is the occurrence of inter-user interference, i.e. nonzero response of the receiver of the k-th user to the signals of other users [2].

Based on this, it is necessary to choose such code sequences, in which the CCF is minimal. Moreover, for solving the problem of asynchronous reception, only the correlation properties of the signal are important, but not their shape.

For broadband communication systems, it is necessary to take into account another important parameter for the PRS - the autocorrelation function (ACF). In general, ACF is defined as:

$$R_u(t_o) = \int_{-\infty}^{\infty} u(t)u(t - t_o)dt. \qquad (3)$$

The ACF of phase-modulated signals consists of a central (main) peak located in the interval $(-t_o, t_o)$ and side (background) maxima, which are distributed in the interval (-T, t_o) and (t_o, T). The amplitudes of the side peaks take different values, but for signals with a "good" correlation, they should be minimal, i.e. significantly less than the amplitude of the central peak. The ratio of the amplitude of the central peak to the maximum amplitude of the lateral maxima is called the suppression coefficient.

Discrete signals with the best ACF structure include Barker signals (codes) [5]. The code sequence of the Barker signal consists of N symbols ± 1 and is characterized by a normalized ACF of the form:

$$R_u(n) = \begin{cases} 1, \text{ для } n = 0, \\ 0, \text{для} n = 2l + 1, \\ \pm 1/N, \text{для } n = 2l, \end{cases} \qquad (4)$$

where l = 0, 1, ... (N-1) / 2.

The sign in the last line depends on the value of N. These signals have a unique property: regardless of the number of positions N in the code combination, the ACF values calculated by formula (4) do not exceed one for all $n \neq 0$. At the same time, the energy of these signals, i.e. the value R_u (0), numerically equal to N. Only seven Barker signals are known, the most complex of which consists of 13 symbols and has a suppression coefficient equal to 13. This property makes it possible to reliably detect such a signal when the signal-to-noise ratio $P_s/ P_n<1$.

However, in data transmission channels where there is significant interference, even Barker signals do not provide the required reliability of their detection. In [5], a method for forming composite Barker codes with correlation properties similar to those of the Barker code is proposed, namely: this code is formed by multiplying two standard Barker sequences. One of them (the short one) is called the generative one, and the second, longer one is called the elementary one. As a result of multiplying a short sequence by a longer one, sequences of more than 13 digits are obtained. Such sequences are called direct composite Barker codes.

In [6], it is proposed to form a composite code in another way, namely, by multiplying a long sequence by a short one, i.e., to use longer sequences as a generator, and short Barker sequences as an elementary one. Let's call these sequences inverse composite Barker codes.

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The paper analyzes direct and inverse composite Barker codes based on the canonical sequences C4 and C7. Table 1 shows the formation of direct and inverse composite Barker codes with a length of 28 characters based on the canonical sequences C4 and C7.

Table 1: Direct and inverse composite Barker codes

Barker code length	Code charactervalues
$C4_1$	+1+1+1-1
$C4_2$	+1+1-1+1
C7	+1 +1 +1 -1 -1 +1 -1
$C4_1 \cdot C7$	+1+1+1-1-1+1-1+1+1+1-1-1+1-1+1+1+1-1-1+1-1-1-1-1+1+1-1+1
$C4_2 \cdot C7$	+1+1+1-1-1+1-1+1+1+1-1-1+1-1-1-1-1+1+1-1+1+1+1+1-1-1+1-1
$C7 \cdot C4_1$	+1+1+1-1+1+1+1-1+1+1+1-1-1-1-1+1-1-1-1+1+1+1+1-1-1-1-1+1
$C7 \cdot C4_2$	+1+1-1+1+1+1-1+1+1+1-1+1-1-1+1-1-1-1+1-1+1+1-1+1-1-1+1-1

In addition, to reduce the level of multiple access interference, the applied PRSs should be orthogonal, since in this case the CCF between any other PRSs will be equal to zero. Codes with suchproperties include Haar, Rademacher, Walsh codes[7]. However, a feature of orthogonal codes is that the orthogonality of these codes is performed only at the "point", i.e., in the absence of time shifts. In reality such conditions are not met, orthogonality is violated, which in turn leads to an increase of the level of multiple access interference and to the appearance of errors in the input data processing. Therefore, various methods are used to eliminate these disadvantages.

Let us consider the application of the so-called composite Walsh codes from this point of view. Such codes are obtained by multiplying two standard sequences. The first of them (for example, the Walsh function) is called a generator, and the second (for example, M is a sequence) is called elementary. Each character (symbol) in the generating sequence is replaced by a direct or inverse elementary sequence, depending on what value the character has (+1 or -1) in the generating sequence.

As a result of multiplying the first sequence by the second, sequences with a large number of digits are obtained. We call such a sequence the direct composite Walsh function. If as the first is taken M-sequence, and as an elementary one - the Walsh function, then such a sequence will be called the inverse composite Walsh function.

As an M - sequence, we take a primitive, irreducible polynomial of the 3rd order - $h(x) = x^3 + x + 1$, with the help of which we can obtain several PRSs depending on the initial state of the shift register. The most important feature of M - sequences is that their periodic autocorrelation function is optimal in the class of possible autocorrelation functions of binary sequences of length $L = 2^n - 1$. Just due to good autocorrelation properties of M - sequences and the simplicity of their formation they are widely used in communication systems [1-4].

As the Walsh function, we choose the 4th order Walsh function, which can take the following values: {(+1, +1, +1, +1), (+1, +1, -1, -1), (+1, -1, -1, +1), (+1, -1, 1, -1}. In this work is considered the following M - sequence – (+1, -1, -1, +1, +1, +1, -1).

Using it as a generating or elementary sequence, we form the direct and inverse composite Walsh functions.

III. SIMULATION & RESULTS

Analysis of the correlation characteristics of direct $C4_1 \cdot C7$, $C4_2 \cdot C7$ and inverse $C7 \cdot C4_1$, $C7 \cdot C4_2$ composite Barker codes, performed by simulation in the Matlab environment, gave the results shown in figure. 1.1 – figure.1.6.

Analysis of the correlation characteristics of direct and inverse composite Walsh functions, which were mentioned above, carried out by simulation in the Matlab environment, gave the results shown in figure. 1.7 - figure. 1.12.

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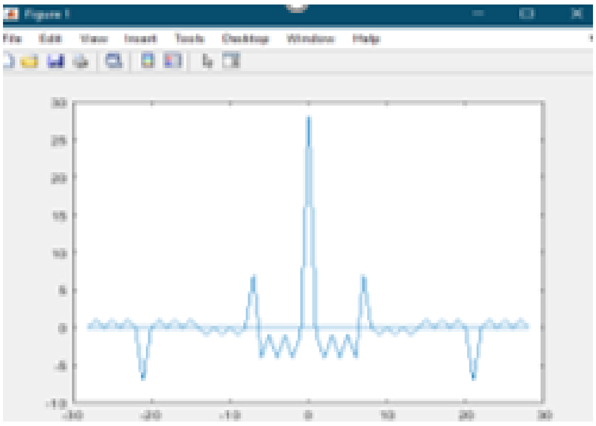

Figure 1: ACF of direct composite Barker code $C4_1 \cdot C7$

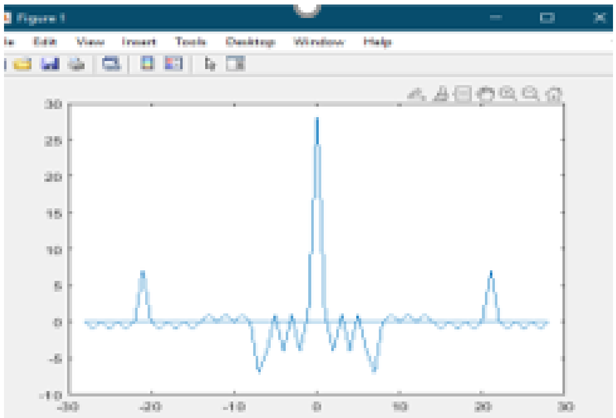

Figure 2: ACF of direct composite Barker code$C4_2 \cdot C7$

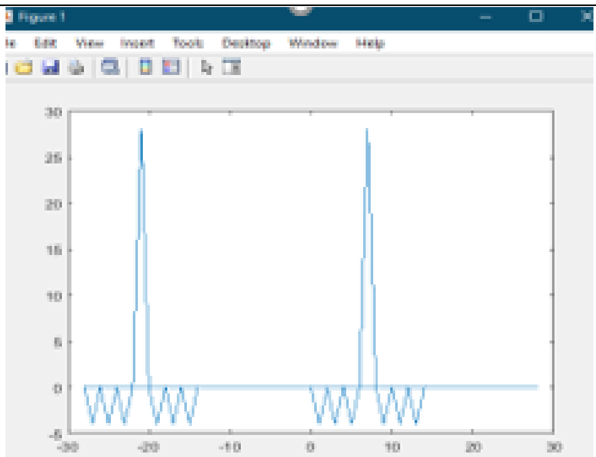

Figure 3: CCF of direct composite Barker codes

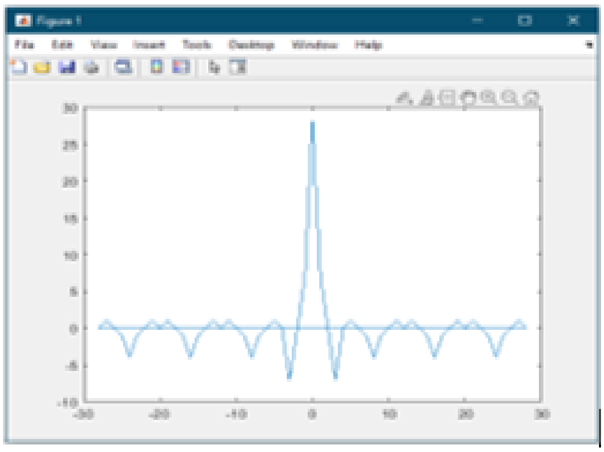

Figure 4: ACF of inverse composite Barker code$C7 \cdot C4_1$

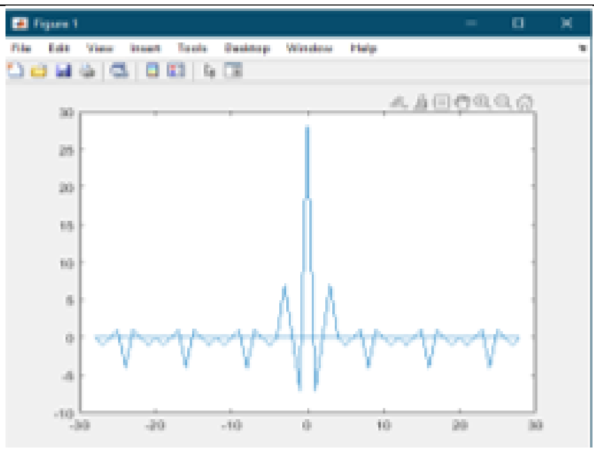

Figure 5: ACF of inverse composite Barker code $C7 \cdot C4_2$

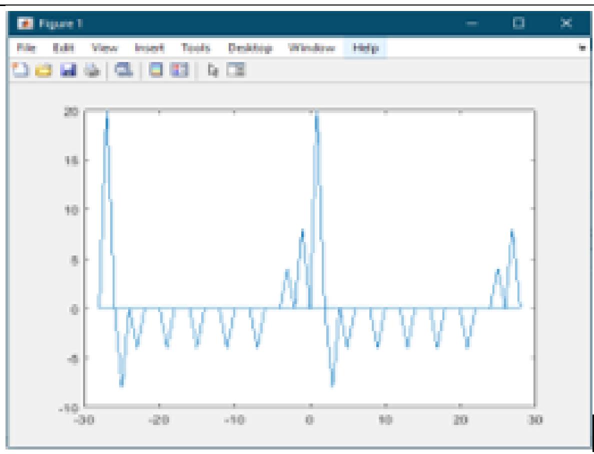

Figure 6: CCF of inverse composite Barker codes

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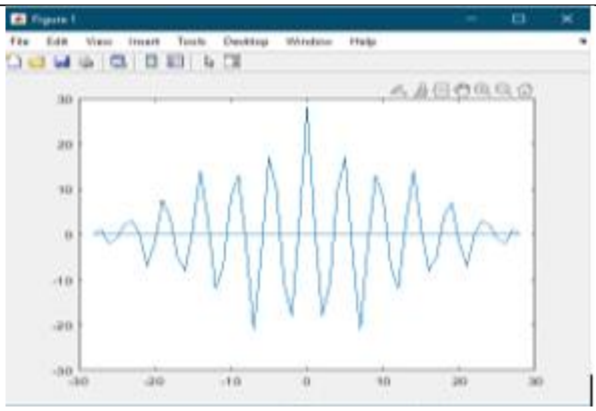

Figure 7: ACF of direct composite Walsh functions and M – sequences: (1, -1,1, -1) *(1, -1, -1, 1, 1, 1, -1)

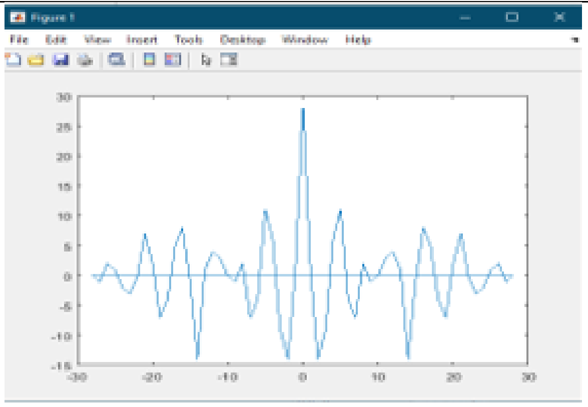

Figure 8: ACF of direct composite Walsh functionsand M – sequences: ((1, -1, -1,1)) *(1, -1, -1,1,1,1, -1)

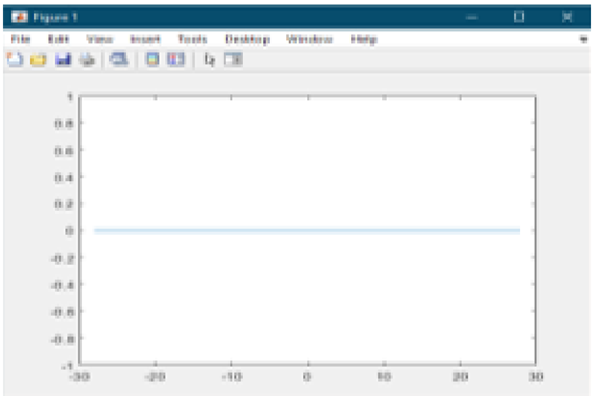

Figure 9: CCF of direct composite Walsh functions

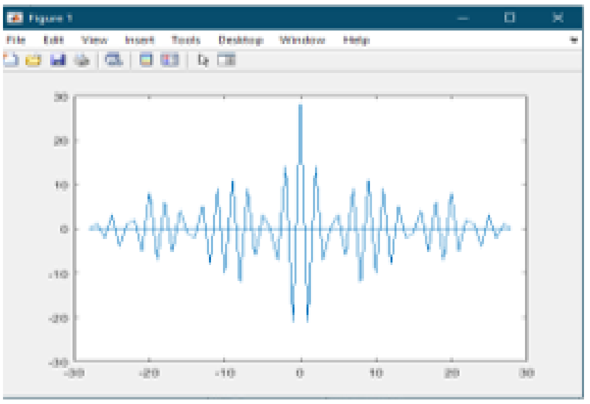

Figure 10: ACF of inverse composite Walsh functions and M -sequences: (1, -1, -1, 1, 1, 1, -1) * (1, -1, 1, -1)

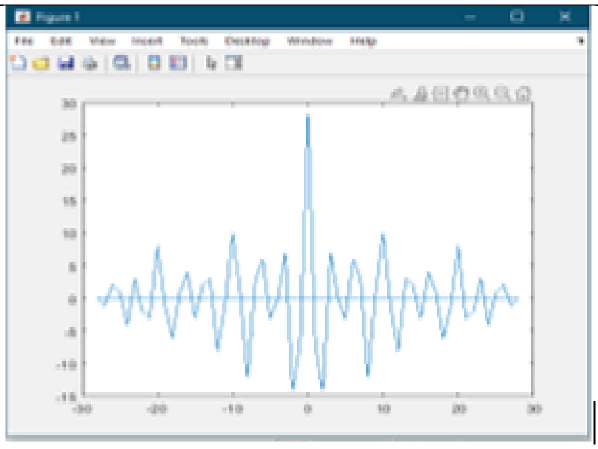

Figure 11: ACF of inverse composite Walsh functions and M -sequences:(1,1,1,1,1,1,1) *(1,1,1,1)

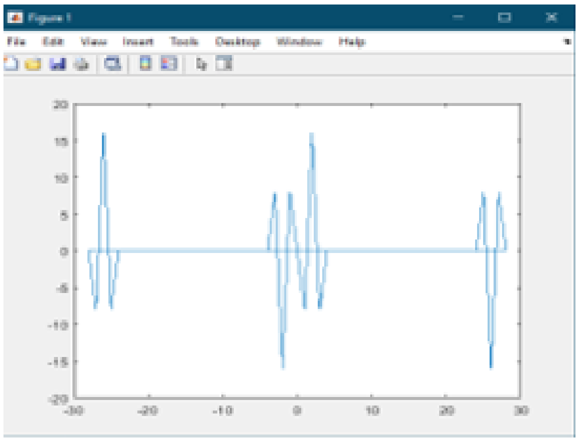

Figure 12: CCF of inverse composite Walsh functions

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IV. DISCUSSIONS

Analysis of the correlation characteristics of direct $C4_1 \cdot C7$, $C4_2 \cdot C7$ and inverse $C7 \cdot C4_1$, $C7 \cdot C4_2$ composite Barker codes showed that they have approximately the same correlation characteristics, the suppression coefficient of both PRSs is the same, equal to 4 and can be used in broadband data transmission systems. The large length of the extension code (L = 28) allows to increase significantly the energy dissipation of the transmitter over the bandwidth of the communication channel, increase the noise immunity of the system, good protection against unauthorized access, and improve electromagnetic compatibility with neighboring operating radio equipment.

However, for code division multiplexing systems, it is necessary to have a PRS ensemble with “good” auto- and cross-correlation characteristics. Various modifications of Barker codes do not make it possible to obtain such ensembles, as can be seen from Fig.3 and Fig.6, direct and inverse composite Barker codes have poor cross-correlation characteristics and are undesirable to use in code division systems due to the significant level of multiple access interference.

From figure 7 –figure 9 it follows that the direct, composite Walsh functions based on M - sequences have poor autocorrelation functions - there is no pronounced central lobe of the characteristic and large levels of side lobes, the suppression coefficient is 2. But they have very good cross correlation characteristics. Thus, direct composite Walsh functions cannot provide reliable synchronization during reception, correct decoding of input data, but provide a minimum level of multiple access interference, since the cross-correlation function is zero at any time offsets.

From figure 10 -figure 12 it follows that the inverse composite Walsh functions have better autocorrelation functions compared to the direct ones - there is a pronounced central lobe of the characteristic and lower levels of side lobes, but not very good cross-correlation characteristics. Thus, the inverse composite Walsh functions can provide reliable timing on reception, correct decoding of the input data, but slightly increase the level of multiple access interference at small timeshifts, and much less at large time shifts.

V. CONCLUSION

Analysis of the correlation characteristics of direct and inverse composite Barker codes and direct and inverse composite Walsh functions allows to draw the following conclusions:

1. Composite Barker codes have better autocorrelation characteristics than composite Walsh functions. Their suppression rate is twice as high. This allows them to be used for synchronization during reception.
2. Composite Barker codes have poor cross-correlation characteristics, which do not allow them to be used in systems with code division channels.
3. Direct composite Walsh functions are time-shift invariant, i.e., the cross-correlation of such functions is independent of the time shift and is orthogonal. This allows them to be used in systems with code separation channels, but it is necessary to apply measures to improve synchronization during reception.
4. The type of correlation characteristics of composite Walsh functions largely depends on the type of generators and elementary sequences that are conjugated with the corresponding Walsh functions.

The area of application of PRS, which are based on various approaches, is wide and varied. By selecting the appropriate properties of the generators or elementary sequences, a satisfactory result can be achieved in most cases of operation of broadband systems. Generation of PRS ensembles of arbitrary length with given correlation properties is an urgent practical problem.

Consequently, further careful study of the properties of derivatives and elementary sequences is required for the formation of various composite functions for solving the corresponding applied problems.

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