ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 3 , March 2021

Algorithms for determining the placement of poles in multivariate systems with proportional-differential output feedback

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ABSTRACT: Algorithms for determining the placement of poles in multivariate systems using proportional-differential output feedback. The requirements for the system are formulated by specifying the desired distribution on the complex plane of the eigenvalues of the matrix of the closed-loop system. Two possible approaches to determining the values of the matrix are considered, using proportional feedback. The proposed approaches to solving the problems posed are based on the boundaries of the total number of poles. The main focus was on finding the value of the feedback matrix using two search approaches. The above algorithm provided an effective solution to the problem of pole placement in process control systems.

KEY WORDS: linear system, modal control, pole placement, feedback matrices.

I. INTRODUCTION

One of the most common synthesis methods based on representing systems in the state space is the modal control method. In comparison with the optimal synthesis, the modal synthesis method has such advantages as simplicity, formalization, as well as the solution of the problem of smoothness of the transient process and increasing the speed of response.

The modal control problem, which consists in assigning the given roots of the characteristic equation of closed linear systems using linear feedback, has been sufficiently studied and is widely used in control theory and practice [1-8]. It is known [1,5] that in the case when the number of inputs of the control object is more than one, the modal control problem has an infinite set of solutions in the sense of multivariance in the choice of feedback matrices. In the problem of modal control, the eigenvalues of a closed-loop system are assumed to be given, and the emerging freedom of choice of elements of the feedback matrix in systems with many inputs can be used only to vary the matrix of eigenvectors of the closed-loop system.

It is known [1,4,5] that it is possible to obtain any spectrum of a linear system with the help of rigid feedback if this system is completely controllable.

If for a completely controllable system the number of nontrivial invariant polynomials of the matrix of the system is less than the dimension of the control vector, then the problem of constructing a control leading to the required spectrum is not uniquely solved. In this case, it is of interest to synthesize a system with a given or satisfying some requirements spectrum under conditions of incomplete information required for the synthesis of a system with the required spectral properties [1,8-13].

II. FORMULATION OF THE PROBLEM

Consider a system with one-dimensional input and multidimensional output:

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\},$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
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where $x - n$ is a dimensional state vector, u is a scalar input, $y - l$ is a dimensional vector of output (controlled) variables; A, b, C are matrices.
Determine the number of poles that can be arbitrarily placed in such a system using proportional-differential output feedback [1,5]. The system under consideration can be described using the transfer function:

$$Y(s)=(W(s)/F(s))U(s),$$

where

$$F(s)=|sI-A|=s^n+d_1s^{n-1}+d_2s^{n-2}+...+d_n,$$

$$w(s)=\left[Cb, CAb,.., CA^{n-1}b\right]\begin{bmatrix}1 & d_1 & d_2 & \dots & d_{n-1}\\ 0 & 1 & d_1 & \dots & d_{n-2}\\ \vdots & 0 & 1 & & \vdots\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & & 1\end{bmatrix}\begin{bmatrix}s^{n-1}\\ s^{n-2}\\ \vdots\\ \vdots\\ s^0\end{bmatrix}. \tag{1}$$

Let in the system with the help of l-dimensional vectors p and q is formed proportional-differential feedback on the output. As a result, the characteristic polynomial of the n - th order closed-loop system is determined by the expression:

$$H(s)=\frac{1}{1+qCb}\{F(s)+(p+qs)w(s)\}=\frac{1}{1+qCb}\left\{F(s)+[p,q]\begin{pmatrix}w(s)\\ sw(s)\end{pmatrix}\right\}.$$

Rotation (1) and the Cayley-Hamilton theorem allow us to write $H(s)$ in the form [1,8]:

$$H(s)=\frac{1}{1+[p,q]\begin{pmatrix}0\\ Cb\end{pmatrix}}\left\{[1,d_1,d_2,...,d_n]+[p,q]\begin{pmatrix}0 & Cb & CAb & \dots & CA^{n-1}b\\ Cb & CAb & CA^2b & \dots & CA^nb\end{pmatrix}D\right\}\begin{pmatrix}s^n\\ s^{n-1}\\ \vdots\\ s^0\end{pmatrix}, \tag{2}$$

Where

$$D=\begin{bmatrix}1 & d_1 & \dots & d_{n-1} & d_n\\ 0 & 1 & \dots & d_{n-2} & d_{n-1}\\ \vdots & 0 & \dots & \vdots & \vdots\\ \vdots & \vdots & \dots & 1 & \vdots\\ 0 & 0 & \dots & 0 & 1\end{bmatrix}$$

is a non-degenerate $(n+1)\times(n+1)$ - matrix. Expression (2) shows that the coefficient at s^n in $H(s)$ is always equal to one and does not depend on s^n and $H(s)$. The coefficients at $s^{n-1},..., s^0$ depend on p and q.

The number of coefficients that can be set independently using p and q is equal to the number of linearly independent columns of the matrix:

$$\begin{pmatrix}Cb & CAb & \dots & CA^{n-1}b\\ CAb & CA^2b & & CA^nb\end{pmatrix}.$$

The number of poles in a closed system, which can be arbitrarily set with p and q, is given by:

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International Journal of Advanced Research in Science, Engineering and Technology
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$$v_1 = \operatorname{rank}\begin{pmatrix} Cb & CAb & \dots & CA^{n-1}b \\ CAb & CA^2b & & CA^n b \end{pmatrix} = \operatorname{rank}\left[\begin{pmatrix} C \\ CA \end{pmatrix} \quad R_c\right],$$

where $R_c = (b,\ Ab,\ \dots,\ A^{n-1}b)$ is the controllability matrix of the system.

In the case when the system is fully controllable, $\operatorname{rank} R_c = n$ and

$$v_1 = \operatorname{rank}\begin{pmatrix} C \\ CA \end{pmatrix}.$$

Note that $v_1 \le \min(2l,\ n)$, where $2l$ is the total number of parameters that define the feedback vectors.

Similarly, consider the (A, B, C) system with m inputs and one output. The use of n-dimensional vectors p and q proportional-differential feedback on the output allows you to arbitrarily set [1,8,14]:

$$v_2 = \operatorname{rank}\{R_0^T [B, AB]\},$$

poles of a closed system, where

$$R_0 = \left(c^T, A^T c^T, \dots, \left(A^T\right)^{n-1} c^T\right)$$

is the observability matrix of the system. When the system is fully observable:

$$\operatorname{rank} R_0 = n \text{ и } v_2 = \operatorname{rank}[B, \quad AB].$$

In this case, $v_2 \le \min(2m,\quad n)$, where $2m$ represents the total number of elements of vectors p and q.
We turn to consideration of a controlled, observable linear multidimensional system:

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\},$$

where x – n -dimensional vector of state, u – m -dimensional vector of input, y – l -dimensional vector of output (controlled) variables.

We assume that the system uses a proportional-differential control law in the form of output feedback

$$u(t) = v(t) - Py(t) - Q\dot{y}(t),$$

where v – m is a dimensional setting vector, P and Q are proportional and differential $m \times 1$ are output feedback matrices, respectively. The equation of state of a closed-loop system has the form:

$$\dot{x}(t) = (I + BQC)^{-1}(A - BPC)x(t) + (I + BQC)^{-1} Bv(t),$$

where $|I + BQC| \ne 0$.

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Let us consider two possible approaches to the definition of such feedback matrices P and Q, which provide an arbitrary setting of the poles in a closed system, i.e. eigenvalues of the matrix [1,5]:

$$\hat{A} = (I + BQC)^{-1}(A - BPC).$$

III. SOLUTION OF THE TASK

We will construct feedback matrices in two stages. In the first approach, $m-1$ poles are set in the first step using proportional feedback. Let us introduce into the system (A, B, C) proportional $m \times l$ - the feedback matrix of unit rank $P_1 = k_1 p_1$ to obtain the characteristic polynomial:

$$\mathrm{H}_1(s) = H_0(s) + p_1 W_0(s) k_1,$$

where $W_0(s) = C\,\mathrm{adj}(sI - A)B$, $H_0(s) = |sI - A|$, l -dimensional vector p_1 is defined so that $(A, p_1 C)$ is observable, and m -dimensional vector k_1 is defined so that $m-1$ poles of the closed-loop system take different preset values $\lambda_1, \ldots, \lambda_{m-1}$.

For this, $m-1$ linear equations are solved:

$$H_0(\lambda_i) + p_1 W_0(\lambda_i) k_1 = 0, \quad i = 1, \ldots, m-1.$$

As a result, the closed system (A_1, B, C), where $A_1 = A - BP_1C$, has $m-1$ poles with values of $\lambda_1, \ldots, \lambda_{m-1}$. At the second stage, we introduce into the system (A_1, Bk, C) proportional and differential $m \times 1$ - feedback matrices of unit rank $P_2 = kp_2$ and $Q = kq$, respectively, where k is m -dimensional vector, p_2 and q are l -dimensional vectors.

Thus, the problem is reduced to a system (A_1, Bk, C) having a one-dimensional input, a vector of proportional feedback P_2 and a vector of feedback on the derivative q. The characteristic polynomial of a closed-loop system is determined by the expression:

$$H_2(s) = \frac{1}{1 + q\,CBk}\left[H_1(s) + p_2 W_1(s) k + sqW_1(s) k\right],$$

where $W_1(s) = C\,\mathrm{adj}(sI - A_1)B$, a $H_1(s) = |sI - A_1|$.

The vector k is necessary to preserve the values of $m-1$ poles of system (A_1, B, C) in a closed system regardless of p_2 and q. For this, the following conditions must be met:

$$W_1(\lambda_i) k = 0, \; i = 1, \ldots, m-1.$$

Since the matrices $\mathrm{adj}(\lambda_i I - A_1)$, $i = 1, \ldots, m-1$ are rank one, each of the matrices $W_1(\lambda_i)$ contains only one independent row w_i.

Therefore, the vector k is determined from $m-1$ linear equations:

$$w_i k = 0, \; i = 1, \ldots, m-1.$$

The number of poles that can be set arbitrarily in the (A_1, Bk, C) system with one-dimensional input using the feedback vectors p_2 and q is determined by the expression:

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International Journal of Advanced Research in Science, Engineering and Technology
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$$v_1 = \text{rank}\left[\begin{pmatrix} C \\ CA_1 \end{pmatrix} \quad R_C\right],$$

where $R_C = \left(Bk,\ A_1 Bk, \ldots, A_1^{n-1} Bk\right)$ is the controllability matrix for system $\left(A_1,\ Bk, C\right)$. Vectors p_2 and q providing additional poles with values of v_1 are obtained by solving $\lambda_m, \ldots, \lambda_{m+v_1-1}$ linear λ_1 equations:

$$H_1(\lambda_i) + p_2 W_1(\lambda_i)k + \lambda_i q W_1(\lambda_i)k = 0,\ i = m,\ \ldots,\ m + v_1 - 1.$$

Thus, the proportional feedback matrix:

$$P = \left(k_1, \quad k\right)\begin{pmatrix} p_1 \\ p_2 \end{pmatrix},$$

whose rank is equal to 2 and the differential feedback matrix $Q = kq$ of unit rank set $\rho_1 = m + v_1 - 1$ values of the poles of the system $\left(A,\ B,\ C\right)$.

It can be shown that with the considered approach, the boundaries of the total number of poles, specified using proportional-differential feedback, are determined by the inequality:

$$\alpha \le \rho_1 \le \gamma_1 - 1,$$

where $\alpha = \text{rank}\begin{pmatrix} C \\ CA \end{pmatrix}$, a $\gamma_1 = m + \min\left(\alpha, \quad n - m + 1\right)$.

Note that $\rho_1 \le 2l + m - 1$.

In the second approach, in the first step, the values of $l-1$ poles are set using proportional feedback. Let us introduce into the system $\left(A,\ B,\ C\right)$ a proportional feedback matrix of unit rank $P_1 = k_1 p_1$ to set the $l-1$ poles of the values $\lambda_1, \ldots, \lambda_{l-1}$. In this case, K_1 is arbitrary, and P_1 is determined from the solution of $l-1$ linear equations:

$$H_0(\lambda_i) + p_1 W_0(\lambda_i)k_1 = 0, \quad i = 1, \ldots, l-1,$$

where $W_0(s) = C\,\text{adj}(sI - A)B$, a $H_0(s) = |sI - A|$.

At the second stage, we enter $\left(A,\ B,\ C\right)$ into the system, where $A_1 = A - BP_1C$, proportional and differential feedback matrices of unit rank $P_2 = pk_2$ and $Q = qk$, respectively. Vector k is l-dimensional, and vectors p_2 and q are m-dimensional.

Thus, the problem is reduced to a system $\left(A_1,\ B,\ kC\right)$ with a one-dimensional output, proportional to the feedback vector p_2 and the vector feedback from the derivative q. The characteristic polynomial of a closed system is determined by the expression:

$$H_2(s) = \frac{1}{1 + kCBq}\left[H_1(s) + kW_1(s)p_2 + skW_1(s)q\right],$$

where $W_1(s) = C\,\text{adj}(sI - A_1)B$, a $H_1(s) = |sI - A_1|$.

Vector k is necessary to store the values $l-1$ of the poles of system $\left(A_1,\ B,\ C\right)$ in a closed system, regardless of p_2 and q. For this, the following conditions must be met:

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International Journal of Advanced Research in Science, Engineering and Technology
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$$kw_i = 0\,,\quad i = 1,\ldots,l-1,$$

where w_i represents the non-zero column $W_1(\lambda_i)$.
The number of poles that can be set arbitrarily in the system $(A_1,\,B,\,kC)$ with one-dimensional output using the feedback vectors p_2 and q is determined by the value

$$v_2 = \operatorname{rank}\left\{R_0^T\,[B, A_1B]\right\},$$

where $R_0 = \left[(kC)^T, A_1^T(kC)^T, \ldots, \left(A_1^T\right)^{n-1}(kC)^T\right]$ is the observability matrix for system $(A_1,\,B,\,kC)$.
Vectors p_2 and q, providing the assignment of 333 additional poles of values, are obtained by solving $\lambda_l,\ldots,\lambda_{l+v_2-1}$ linear v_2 equations:

$$H_1(\lambda_i) + kW_1(\lambda_i)p_2 + \lambda_i kW_1(\lambda_i)q = 0\,,\qquad i = l,\,\ldots,\,l+v_2-1\,.$$

Thus, the proportional feedback matrix $P = P_1 + P_2$, which has a rank of two, and the differential feedback matrix Q of rank one, define $\rho_2 = l + v_2 - 1$ pole values of the system $(A,\,B,\,C)$.
It can be shown that

$$\beta \le \rho_2 \le \gamma_2 - 1\,,$$

where $\beta = \operatorname{rank}[B, AB]$, a $\gamma_2 = l + \min(\beta,\quad n-l+1)$.
Note that $\rho_2 \le 2m + l - 1$.

IV. CONCLUSION

In conclusion, we note that in the multidimensional system $(A,\,B,\,C)$, the number of poles specified using the proportional feedback matrix P and the differential feedback matrix Q is determined by the number $\rho = \max(\rho_1, \rho_2)$, the boundaries of which are determined by the inequality:

$$\max(\alpha, \beta) \le \rho \le \max(\gamma_1, \gamma_2) - 1.$$

Note that $\rho \le \max(2m + l - 1,\quad 2l + m - 1)$.
When determining the required matrices P and Q, the first approach should be used when $\rho_1 \ge \rho_2$, and the second approach when $\rho_1 < \rho_2$.
The results of the analysis have confirmed their effectiveness, which makes it possible to use them in solving applied problems of optimizing the parameters of controlled systems and synthesizing control systems for technological processes.

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