ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 5 , May 2021

Arithmetic-Harmonic Mean: A Measure of Central Tendency of Ratio-Type Data

Dhritikesh Chakrabarty

Associate Professor, Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India

ABSTRACT: Recently one formulation of average namely Arithmetic-Harmonic Mean (abbreviated as *AHM*) has been developed along with three more formulations namely Arithmetic-Geometric Mean (abbreviated as *AGM*), Geometric-Harmonic Mean (abbreviated as *GHM*) and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) from the three Pythagorean means namely Arithmetic Mean (*AM*), Geometric Mean (*GM*) and Harmonic Mean (*HM*). Each of these four formulations can be a measure of central tendency of data in addition to the three popular measures of central tendency namely *AM*, *GM* & *HM*. This paper describes that *AHM* can be a suitable measure of central tendency of numerical data of ratio type. The associated theoretical description/explanation along with numerical application has been presented in this paper.

KEYWORDS: Arithmetic-Harmonic Mean, Ratio-Type Data, Central Tendency, Measure

I. INTRODUCTION

A number of researches had already been done on developing definitions / formulations of average [1 , 2], a basic concept used in developing most of the measures used in analysis of data. Pythagoras [3], the pioneer of researchers in this area, constructed three definitions / formulations of average namely Arithmetic Mean (abbreviated as *AM*), Geometric Mean (abbreviated as *GM*) & Harmonic Mean (abbreviated as *HM*) which are called Pythagorean means as a mark of respect to Pythagoras [4 , 5 , 14 , 18]. A lot of definitions / formulations in addition to *AM* , *GM* & *HM* have been developed among which some are etc. [6 , 7 , 8 , 9 , 10 , 11 , 12 , 13 , 14 , 15 , 16 , 17, 18 , 19]. Kolmogorov [20] formulated one generalized definition of average namely Generalized f - Mean. [7 , 8]. It has been shown that the definitions/formulations of the existing means and also of some new means can be derived from this Generalized f - Mean [9 , 10]. In an study, Chakrabarty formulated one generalized definition of average namely Generalized f_H – Mean [11]. In another study, Chakrabarty formulated another generalized definition of average namely Generalized f_G – Mean [12 , 13] and developed one general method of defining average [15, 16 , 17] as well as the different formulations of average from the first principles [19].

In many real situations, observed numerical data

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

are found to be composed of a single parameter μ and corresponding chance / random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_N$$

i.e. the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , N) \tag{1.1}$$

[21 , 22 , 23 , 24 , 25 , 26 , 27, 28 , 29].

The existing methods of estimation of the parameter μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [31 , 32 , 33 , 34 , 35, 36 , 37, 38 , 39 , 40 , 41 , 42 , 43 , 44 , 45 , 46 , 47 , 48 , 49 , 50 , 51 , 52] cannot provide appropriate value of the parameter μ [21 , 22 , 23]. In some recent studies, some methods have been developed for determining the value of parameter from observed data containing the parameter itself and random error [21 ,22 , 23 , 24 , 25 , 26, 27 , 28 , 29 , 30 , 53 ,54 , 55 , 56, 57 , 58 , 59 , 60]. The methods, developed in this studies, for determining the appropriate value of the parameter from observed data containing the parameter itself and random error involve huge computational tasks. Moreover, a finite set of observed data may not yield the appropriate value of the parameter in many situations while the number of observations required in the methods may be too large for obtaining the appropriate value of the parameter. However, the appropriate value of the parameter is not perfectly attainable in practical situation. What one can expect is to obtain that value which is more and more close to the appropriate value of the parameter. In order to

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 5 , May 2021

obtain such value of parameter, four methods have already been developed which involves lesser computational tasks than those involved in the earlier methods as well as which can be applicable in the case of finite set of data [61 , 62 , 63 , 64]. The methods developed are based on the concepts of Arithmetic-Geometric Mean (abbreviated as *AGM*) [61 , 62, 67 , 68 . 69 , 70], Arithmetic-Harmonic Mean (abbreviated as *AHM*) [63], Geometric-Harmonic Mean (abbreviated as *GHM*) [64] and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) [65, 66 , 67] respectively.
Each of these four formulations can be a measure of central tendency of data in addition to the three popular measures of central tendency namely *AM*, *GM* & *HM*. This paper describes that *AHM* can be a suitable measure of central tendency of numerical data of ratio type. The associated theoretical description/explanation along with numerical application has been presented in this paper.

II. FORMULATIONS OF *AGM, AHM, GHM & AGHM*

Let a_0 , g_0 & h_0 be respectively the *AM*, the *GM* & the *HM* of N positive numbers or values or observations (not all equal or identical)

$$x_1 , x_2 , \ldots\ldots , x_N$$

all of which are not equal i.e.

$$a_0 = AM(x_1 , x_2 , \ldots\ldots , x_N) = \frac{1}{N}\sum_{i=1}^{N} x_i ,$$
$$g_0 = GM(x_1 , x_2 , \ldots\ldots , x_N) = \left(\prod_{i=1}^{N} x_i\right)^{1/N}$$
$$\& \; h_0 = HM(x_1 , x_2 , \ldots\ldots , x_N) = \left(\frac{1}{N}\sum_{i=1}^{N} x_i^{-1}\right)^{-1}$$

Then a_0 , g_0 & h_0 satisfy the Pythagorean Inequality [4 , 5] namely

$$AM > GM > HM \quad \text{i.e.} \quad h_0 < g_0 < a_0 \qquad (2.1)$$

ARITHMETIC-GEOMETRIC (AGM)

The two sequences $\{a_n\}$ & $\{g_n\}$ respectively defined by

$$a_{n+1} = ½\, a_n + g_n) ,$$
$$\& \quad g_{n+1} = (a_n\, g_n)^{1/2}$$

where the square root takes the principal value,
converge to a common point (real number) M_{AG}
which can be termed as the Arithmetic-Geometric Mean (abbreviated as *AGM*) of the *N* values $x_1 , x_2 , \ldots\ldots , x_N$ [61 , 62, 66 , 67 , 68] i.e.

$$AGM(x_1 , x_2 , \ldots\ldots , x_N) = M_{AG}$$

ARITHMETIC-HARMONIC MEAN (AHM)

Let $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{h'_n = h'_n(a_0 , h_0)\}$ be two sequences defined by

$$a'_{n+1} = ½(a'_n + h'_n)$$
$$\& \quad h'_{n+1} = ½(a'^{-1}_n + h'^{-1}_n)\}^{-1}$$

respectively.
Then, the two sequences $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{h'_n = h'_n(a_0 , h_0)\}$
converge to a common point (real number) M_{AH}
which can be termed the Arithmetic-Harmonic Mean (abbreviated as *AHM*) of $x_1 , x_2 , \ldots\ldots , x_N$ [63 , 66] i.e.

$$AHM(x_1 , x_2 , \ldots\ldots , x_N) = M_{AH}$$

GEOMETRIC-HARMONIC MEAN (GHM)

The two sequences $\{g''_n\}$ & $\{h''_n\}$ defined respectively by

$$g''_{n+1} = (g''_n \,.\, h''_n)^{1/2}$$
$$\& \quad h''_{n+1} = \{½(g''^{-1}_n + h''^{-1}_n)\}^{-1}$$

where the square root takes the principal value,
converge to a common point (real number) M_{GH} .
This common converging point M_{GH} can be termed the Geometric-Harmonic Mean (abbreviated as *GHM*) of x_1 , x_2 , $\ldots\ldots$, x_N [64 , 66] i.e.

$$GHM(x_1 , x_2 , \ldots\ldots , x_N) = M_{GH}$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 5 , May 2021

ARITHMETIC-GEOMETRIC-HARMONIC MEAN (AGHM)

The three sequences $\{a'''_n\}$, $\{g'''_n\}$ & $\{h'''_n\}$ defined respectively by

$$a'''_n = 1/3\,(a'''_{n-1} + g'''_{n-1} + h'''_{n-1}) \;,$$
$$g'''_n = (a'''_{n-1}\, g'''_{n-1}\, h'''_{n-1})^{1/3}$$
$$\&\; h'''_n = \{1/3\,({a'''_{n-1}}^{-1} + {g'''_{n-1}}^{-1} + {h'''_{n-1}}^{-1})\}^{-1}$$

converge to a common point (real number) M_{AGH} .
This common converging point M_{AGH} can be termed the Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) of $x_1 , x_2 , \ldots\ldots , x_N$ [65 , 66] i.e.

$$AGHM(x_1 , x_2 , \ldots\ldots , x_N) = M_{AGH}$$

III. AHM AS MEASURE OF CENTRAL TENDENCY OF DATA OF RATIO TYPE

If the observations

$$x_1 , x_2 , \ldots\ldots , x_N$$

be *N* observed values (not all identical) of ratio type.
Then automatically, the values are strictly positive.
Let $\boldsymbol{\mu}$ be the central tendency of the observed values.
Then the observed values can be expressed as

$$x_i = \boldsymbol{\mu} + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , N) \qquad (3.1S)$$

where

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

are the errors associated to

$$x_1 , x_2 , \ldots\ldots , x_N$$

respectively which are random in nature
i.e. each of them assumes either positive real value or negative real value with equal probability.

Again since μ is the central tendency of the observed values

$$x_1 , x_2 , \ldots\ldots , x_N$$

therefore, $\boldsymbol{\mu}^{-1}$ will be the central tendency of reciprocals

$${x_1}^{-1} , {x_2}^{-1} , \ldots\ldots\ldots , {x_N}^{-1}$$

of the observed values.
Accordingly, the reciprocals can be expressed as

$${x_i}^{-1} = \boldsymbol{\mu}^{-1} + \varepsilon_i' \quad , \quad (I = 1 , 2 , \ldots\ldots\ldots , N) \qquad (3.2)$$

where

$$\varepsilon_1' , \varepsilon_2' , \ldots\ldots\ldots\ldots , \varepsilon_N'$$

are the random errors associated to

$${x_1}^{-1} , {x_2}^{-1} , \ldots\ldots\ldots , {x_N}^{-1}$$

respectively which are also random in nature
i.e. each of them assumes either positive real value or negative real value with equal probability.

Let us now write

$$AM(x_1 , x_2 , \ldots\ldots , x_N) = a_0 \qquad (3.3)$$
$$\&\; HM(x_1 , x_2 , \ldots\ldots , x_N) = h_0 \qquad (3.4)$$

and then define the two <u>sequences</u> $\{a'_n\}$ & $\{h'_n\}$ respectively by

$$a'_{n+1} = ½\,(a'_n + h'_n) \qquad (3.5)$$
$$\&\; h'_{n+1} = ½({a'_n}^{-1} + {h'_n}^{-1})\}^{-1} \qquad (3.6)$$

Then, both of $\{a'_n\}$ & $\{h'_n\}$ converges to some common real number C .
Let us now search the relation between C and $\boldsymbol{\mu}$.
Equation (3.1) together with (3.3) & (3.4) implies that

$$a_0 = \boldsymbol{\mu} + \delta_0 \quad \& \quad h_0 = \boldsymbol{\mu} + e_0$$

By inequality (2.4), $h_0 < a_0$ i.e. $e_0 < \delta_0$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 5 , May 2021

Therefore, $a'_1 = \boldsymbol{\mu} + \delta_1$ where $\delta_1 = ½\,(\delta_0 + e_0)$
Since $½\,(\delta_0 + e_0) < \delta_0$
Therefore, $\delta_1 < \delta_0$
At the n^{th} step, one can obtain that

$$\delta_{n+1} = ½\,(\delta_n + e_n) < \delta_n$$

which implies, $\delta_{n+1} < \delta_n$ since $½\,(\delta_n + e_n) < \delta_n$
This implies, δ_n becomes more and more smaller as *n* becomes more and more larger.
This means. a'_n becomes more and more closer to $\boldsymbol{\mu}$ as *n* becomes more and more larger.
Since $\{h'_n\}$ converges to the same point to which $\{a'_n\}$ converges,
Therefore, h'_n also becomes more and more closer to $\boldsymbol{\mu}$ as *n* becomes more and more larger.
Accordingly, the $AHM(x_1, x_2, \ldots\ldots, x_N)$ can be regarded as the value of $\boldsymbol{\mu}$ i.e. the value of the central tendency of $x_1, x_2, \ldots\ldots, x_N$.

IV. NUMERICAL EXAMPLE

Data on the two ratios namely **Male / Female** (abbreviated as **M / F)** & **Female / Male** (abbreviated as **F / M)** of Male and Female in India (state-wise) have been shown in the following table (**Table – 1)**:

Table – 1

State	Value of the Ratio M / F	Value of the Ratio (F / M
Jammu & Kashmir	1.1254138534125111852273651671683	0.88856201384741461016988968870875
Himachal Pradesh	1.0293088804926436613751796256809	0.97152567023553127871119940330966
Punjab	1.11718611741734676457868601138	0.89510600284914783429585712319405
Chandigarh	1.2229968385823537712700642603947	0.81766360177934533455722165869015
Uttarakhand	1.0382445737805593956494862402266	0.96316419584905755859591305415792
Haryana	1.1381499179200197558719403869263	0.878618874592118673847146598073
Delhi	1.1521304409972803426396508480421	0.86795727672502366109786158864161
Rajasthan	1.077386518469311558714857879884	0.92817200035205763708961523638845
Uttar Pradesh	1.0959666766496911194331303675474	0.91243650131493423988837726768373
Bihar	1.0894569681498644304609103396449	0.91788847952225054362107394324387
Sikkim	1.1236943796151050235298618816238	0.88992168879809329247531494722506
Arunachal Pradesh	1.0658345961198241305435082821376	0.93823188292114434272011116216004
Nagaland	1.0742210801874083323111632505218	0.93090707159232088256563955071444
Manipur	1.0150845888535768920299631387912	0.98513957455445833617176866728857
Mizoram	1.0248621894302476437945104610541	0.97574094381990099740878994632108
Tripura	1.0415856043291039214999824955364	0.96007471286444128606000076825568
Meghalaya	1.0113724418785172369610123540989	0.98875543626896326127874988604615
Assam	1.0441048168517855831597956077024	0.95775824788858682201128358123932
West Bengal	1.0526667948213744061675457587868	0.94996821873695430584361430969287
Jharkhand	1.0543346515488809532602154750904	0.9484654597389357492757813425208
Odisha	1.0216767277963741786589610810708	0.97878318336258074151514020087369
Chhattisgarh	1.0094862433659738915914763831542	0.99060289981333128651017560729672
Madhya Pradesh	1.0741921997293521487367677330733	0.93093209972289388478334723747063
Gujarat	1.0878399216924771607664276945985	0.9192528974705997791133158851059
Daman & Diu	1.6170787338884943945947109074086	0.61839907918110990612171575704752
Dadra & Nagar Haveli	1.29217267204182755470193199021	0.77389037985136251032204789430223
Maharashtra	1.0759593940486569345112623151307	0.92940310343605596519523288750508
Andhra Pradesh	1.0072027731513157131279371653056	0.99284873578258743089946488568226
Karnataka	1.0278146308628600560795309711955	0.97293808627776643762353811714322
Goa	1.0274323920462048498411882041409	0.97330005141109938577265470682144
Lakshadweep	1.0565550239234449760765550239234	0.94647223983334842858436735802916

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 5 , May 2021

Kerala	0.92224729321594561234305382426448	1.0843078720382305015931455433978
Tamil Nadu	1.0035802105886977594941050244168	0.99643256159206485698216349975338
Pondicherry	0.96391330758747454527714567183158	1.0374376949964980220763382208646
Andaman & Nicobar	1.1415846041303246862866467840864	0.87597537351321775906857066806002
India	1.0607325851848778252519531570732	0.94274467850509882664736426425148

A. Central Tendency of the Ratio M / F:

From the observed values on the ratio **M / F** in **Table – 3** it has been obtained that
AM of the Ratio **M / F** = 1.0835068016450523020161865887443
& HM of the Ratio **M / F** = 1.0740468088974845410059550737324

The common converging value of the two sequences $\{a'_n\}$ & $\{h'_n\}$, which is the *AHM* of the observed values of the Ratio **M / F**, has been found to be

1.0787664356688097192593273920721

Thus the value of the central tendency of the Ratio **M / F**, obtained by *AHM*, is
1.078766435668809719259327392072

B. Central Tendency of the Ratio F / M:

From the observed values on the Ratio **F / M** in **Table – 3** it has been obtained that
AM of the Ratio **F / M** = 0.9310581175009550726813265197974
& HM of the Ratio **F / M** = 0.92292913942185992242619179784686
The computed values of $\{a'_n\}$ & $\{h'_n\}$,in this case, have been shown in the following table **Table – 3**:
From the observed values on the Ratio **M / F** in **Table – 3** it has been obtained that
AM of **F / M** = 1.0835068016450523020161865887443
& HM of **F / M** = 1.0740468088974845410059550737324

The common converging value of the two sequences $\{a'_n\}$ & $\{h'_n\}$, which is the *AHM* of the observed values of the Ratio **F / M**, has been found to be

0.92698471785509679033872230513345

Thus the value of the central tendency of **F / M**, obtained by *AHM*, is 0.92698471785509679033872230513345.

V. DISCUSSION AND CONCLUSION

If μ is the central tendency of

$$x_1 , x_2 , \ldots\ldots , x_N$$

then the central tendency of

$$x_1^{-1} , x_2^{-1} , \ldots\ldots\ldots , x_N^{-1}$$

should logically be μ^{-1}.
Similarly, the central tendency of

$$-x_1 , -x_2 , \ldots\ldots\ldots , -x_N$$

should logically be $-\mu$.
In the examples, it has been found that the value of central tendency of the ratio **M / F**, obtained by *AHM*, is
1.0787664356688097192593273920721
and the value of central tendency of the ratio **F / M**, obtained by AHM, is
0.92698471785509679033872230513345
These two values are reciprocals each other i.e.
$(1.0787664356688097192593273920721)^{-1} = 0.92698471785509679033872230513345$
& $(0.92698471785509679033872230513345)^{-1} = 1.0787664356688097192593273920721$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 5 , May 2021

Moreover, it is also found that the value of central tendency of the additive inverses of the values of the ratio **M / F**, obtained by *AHM*, is

$$-1.07876643566880971925932739207 21$$

and that of the ratio **F / M** is

$$-0.92698471785509679033872230513345$$

Thus, *AHM* can logically be regarded as an acceptable measure of central tendency of data of ratio type.
It is to be noted that each of *AM* & *HM* does not satisfy these two properties of central tendency and therefore cannot logically be regarded as acceptable measure of central tendency of data of ratio type.
Of course, *GM* satisfies the first property but not the second property of central tendency. Thus, is to be studied further on the acceptability of *GM* as a measure of central tendency of data of ratio type.
Regarding accuracy, it is to be noted that

$$a_0 = \mu + \delta_0 \quad \& \quad \delta_{n+1} < \delta_n$$

This means, δ_n becomes more and more smaller as n becomes more and more larger
which means, a'_n becomes more and more closer to μ as n becomes more and more larger
which further means, A*HM* $(x_1, x_2, \ldots\ldots\ldots, x_N)$ becomes more and more closer to μ as n becomes more and more larger.
Since $\delta_n < \delta_0$ for all $n \geq 1$
therefore, the deviation of A*HM* $(x_1, x_2, \ldots\ldots\ldots, x_N)$ from μ is more than that the deviation of a_0 .
But, a_0 = A*M* $(x_1, x_2, \ldots\ldots\ldots, x_N)$
Hence, A*HM* $(x_1, x_2, \ldots\ldots\ldots, x_N)$ is more accurate measure of central tendency than A*M* $(x_1, x_2, \ldots\ldots\ldots, x_N)$ in the case of data of ratio type.
Similarly, A*HM* $(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be shown to be more accurate measure of central tendency than *HM* $(x_1, x_2, \ldots\ldots\ldots, x_N)$ in the case of data of ratio type.
Therefore, *AHM* can be regarded as a measure of central tendency of data of ratio type which is more accurate than each of *AM* and *HM*. However, it is yet to be studied on the comparison of accuracy of A*HM* with that of *GM* as measure of central tendency of data of ratio type.

It is to be noted that the GM of

AM of the Ratio **M / F** & *HM* of the Ratio **M / F**

is found to be 1.07876643566880971925932739207 21 which is nothing but the AHM of the observed values of the Ratio **M / F**.
Similarly, the GM of

AM of the Ratio **F / M** & *HM* of the Ratio **F / M**

is found to be 0.92698471785509679033872230513345 which is nothing but the *AHM* of the observed values of the Ratio **F / M**.
Thus, *AHM* of the observed values can be regarded as the *GM* of *AM* of the observed values and *HM* of observed values. In general, *AHM*$(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be defined as the *GM* of *AM*$(x_1, x_2, \ldots\ldots\ldots, x_N)$ and *HM*$(x_1, x_2, \ldots\ldots\ldots, x_N)$ in the instant case. However, it is to be established for general case.

On the whole, the two values

1.07876643566880971925932739207 21 and 0.92698471785509679033872230513345

can be regarded as the respective values of central tendency of the Ratio **M / F** and the Ratio **F / M** of the states in India which are very close to the respective actual values while the overall values of these two ratios in India (combing the states) are

1.06073258518487782525195315707 32 and 0.94274467850509882664736426425148

respectively.

However, it is yet to be determined the size of errors or discrepancies in values obtained by *AHM*. It is also to be assessed the performance of *AHM* by applying it in the data with various sample sizes.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 8, Issue 5 , May 2021

REFERENCES

1. Bakker Arthur, "The early history of average values and implications for education", *Journal of Statistics Education*, 2003, 11(1), 17 – 26.
2. Miguel de Carvalho, "Mean, what do you Mean?", *The American Statistician*, 2016, 70, 764 – 776.
3. Christoph Riedweg, "*Pythagoras: his life, teaching, and influence* (translated by Steven Rendall in collaboration with Christoph Riedweg and Andreas Schatzmann, Ithaca)", ISBN 0-8014-4240-0, 2005, Cornell University Press.
4. David W. Cantrell, *"*Pythagorean Means*", Math World*.
5. Dhritikesh Chakrabarty, "Pythagorean Mean: Concept behind the Averages and Lot of Measures of Characteristics of Data", NaSAEAST- 2016, *Abstract ID: CMAST_NaSAEAST (Inv)-1601)*. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
6. Dhritikesh Chakrabarty, "Objectives and Philosophy behind the Construction of Different Types of Measures of Average", NaSAEAST- 2017, *Abstract ID: CMAST_NaSAEAST (Inv)- 1701)*, Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
7. Andrey Kolmogorov , "On the Notion of Mean", in "*Mathematics and Mechanics*" (Kluwer 1991), 1930, 144 – 146.
8. Andrey Kolmogorov, *"Grundbegriffe der Wahrscheinlichkeitsrechnung (in German)*, 1933, Berlin: Julius Springer.
9. Dhritikesh Chakrabarty, "Derivation of Some Formulations of Average from One Technique of Construction of Mean", *American Journal of Mathematical and Computational Sciences*, 2018, 3(3), 62 – 68. Available at http://www.aascit.org/journal/ajmcs.
10. Dhritikesh Chakrabarty, "One Generalized Definition of Average: Derivation of Formulations of Various Means", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN: 2278 – 179 X), 2018, 7(3), 212 – 225. Available at www.jecet.org.
11. Dhritikesh Chakrabarty, "f_H -Mean: One Generalized Definition of Average", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN: 2278 – 179 X), 2018, 7(4), 301 – 314. Available in www.jecet.org.
12. Dhritikesh Chakrabarty, "Generalized f_G - Mean: Derivation of Various Formulations of Average", *American Journal of Computation, Communication and Control*, 2018, 5(3), 101 – 108. Available at http://www.aascit.org/journal/ajmcs .
13. Dhritikesh Chakrabarty, "One Definition of Generalized f_G - Mean: Derivation of Various Formulations of Average", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E- ISSN : 2278 – 179 X), 2019, 8(2), 051 – 066. Available at www.jecet.org.
14. Dhritikesh Chakrabarty, "Pythagorean Mean: Concept behind the Averages and Lot of Measures of Characteristics of Data", NaSAEAST- 2016, *Abstract ID: CMAST_NaSAEAST (Inv)-1601)*, 2016. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
15. Dhritikesh Chakrabarty, "General Technique of Defining Average", NaSAEAST- 2018, *Abstract ID: CMAST_NaSAEAST -1801 (I)*, Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
16. Dhritikesh Chakrabarty, "One General Method of Defining Average: Derivation of Definitions/Formulations of Various Means", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 2019, 8(4), 327 – 338. Available at www.jecet.org .
17. Dhritikesh Chakrabarty, "A General Method of Defining Average of Function of a Set of Values", *Aryabhatta Journal of Mathematics & Informatics* {ISSN (Print) : 0975-7139, ISSN (Online) : 2394-9309}, 2019, 11(2), 269 – 284. Available at www.abjni.com .
18. Dhritikesh Chakrabarty, "Pythagorean Geometric Mean: Measure of Relative Change in a Group of Variables", NaSAEAST- 2019, *Abstract ID: CMAST_NaSAEAST -1902 (I)*, 2019. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
19. Dhritikesh Chakrabarty, "Definition / Formulation of Average from First Principle", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 2020, 9(2), 151 – 163. Available at www.jecet.org .
20. A. P. Youschkevitch , *"*A. N. Kolmogorov: Historian and philosopher of mathematics on the occasion of his 80th birfhday*", Historia Mathematica*, 1983, 10(4), 383 – 395.
21. Dhritikesh Chakrabarty, "Determination of Parameter from Observations Composed of Itself and Errors", *International Journal of Engineering Science and Innovative Technology*, (ISSN: 2139 – 5967), 2014, 3(2), 304 – 311. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
22. Dhritikesh Chakrabarty, "Analysis of Errors Associated to Observations of Measurement Type", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2014, 1(1), 15 – 28. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
23. Dhritikesh Chakrabarty, "Observation Composed of a Parameter and Random Error: An Analytical Method of Determining the Parameter", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2014, 1(2), 20 – 38, 2014. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
24. Dhritikesh Chakrabarty, "Observation Consisting of Parameter and Error: Determination of Parameter", *Proceedings of the World Congress on Engineering*, 2015, ISBN: 978-988-14047-0-1, ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online). Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
25. Dhritikesh Chakrabarty: "Observation Consisting of Parameter and Error: Determination of Parameter," *Lecture Notes in Engineering and Computer Science* (ISBN: 978-988-14047-0-1), London, 2015, 680 – 684. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
26. Dhritikesh Chakrabarty, "Central Tendency of Annual Extremum of Surface Air Temperature at Guwahati", *J. Chem. Bio. Phy. Sci.* (E- ISSN : 2249 – 1929), Sec. C, 2015, 5(3), 2863 – 2877. Available at: www.jcbsc.org.
27. Dhritikesh Chakrabarty, :Central Tendency of Annual Extremum of Surface Air Temperature at Guwahati Based on Midrange and Median", *J. Chem. Bio. Phy. Sci.* (E- ISSN : 2249 –1929), Sec. D, 2015, 5(3), 3193 – 3204. Available at: www.jcbsc.org.
28. Dhritikesh Chakrabarty, "Observation Composed of a Parameter and Random Error: Determining the Parameter as Stable Mid Range", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2015, 2(1), 35 – 47. Available at *http://eses.net.in/ESES Journal*.
29. Dhritikesh Chakrabarty, "A Method of Finding Appropriate value of Parameter from Observation Containing Itself and Random Error", *Indian Journal of Scientific Research and Technology*, (E-ISSN: 2321-9262), 2015, 3(4), 14 – 21. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
30. Dhritikesh Chakrabarty, "Theoretical Model Modified For Observed Data: Error Estimation Associated To Parameter", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2015, 2(2), 29 – 45. Available at *http://eses.net.in/ESES Journal*.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 8, Issue 5 , May 2021

31. Anders Hald, "On the History of Maximum Likelihood in Relation to Inverse Probability and Least Squares", *Statistical Science*, 1999, 14, 214 – 222.
32. Barnard G. A., "Statistical Inference", *Journal of the Royal Statistical Society, Series B*, 1949, 11, 115 – 149.
33. Birnbaum Allan, "On the Foundations of Statistical Inference", *Journal of the American Statistical Association*, 1962, 57, 269 – 306.
34. Ivory, "On the Method of Least Squares", *Phil. Mag*., 1825, LXV, 3 – 10.
35. Kendall M. G. and Stuart A, "*Advanced Theory of Statistics*", *Vol. 1 & 2, 4th Edition, New York, Hafner Press*, 1977.
36. Lehmann Erich L. & Casella George, *Theory of Point Estimation, 2nd ed. Springer.* ISBN 0 – 387 – 98502 – 6, 1998.
37. Lucien Le Cam, "Maximum likelihood — An introduction", *ISI Review,* 1990, 8 (2), 153 – 171.
38. Walker Helen M. & Lev J., "*Statistical Inference*", Oxford & IBH Publishing Company, 1965.
39. Dhritikesh Chakrabarty & Atwar Rahman, "Exponential Curve : Estimation Using the Just Preceding Observation in Fitted Curve", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2007, 3(2), 381 – 386. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
40. Dhritikesh Chakrabarty & Atwar Rahman, "Gompartz Curve : Estimation Using the Just Preceding Observation in Fitted Curve", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2008, 4(2), 421 – 424. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
41. Atwar Rahman & Dhritikesh Chakrabarty, "Linear Curve : A Simpler Method of Obtaining Least squares Estimates of Parameters", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2009, 5(2), 415 – 424. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
42. Dhritikesh Chakrabarty, "Finite Difference Calculus: Method of Determining Least Squares Estimates", *AryaBhatta J. Math. &Info.* (ISSN : 0975 – 7139), 2011, 3(2), 363 – 373. Available at www.abjni.com. Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
43. Atwar Rahman & Dhritikesh Chakrabarty, "General Linear Curve : A Simpler Method of Obtaining Least squares Estimates of Parameters", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2011, 7(2), 429 – 440. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats
44. Dhritikesh Chakrabarty, "Curve Fitting: Step-Wise Least Squares Method", *AryaBhatta J. Math. &Info.*, 2014, 6(1), 15 – 24. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
45. Atwar Rahman & Dhritikesh Chakrabarty, "Elimination of Parameters and Principle of Least Squares: Fitting of Linear Curve to Average Minimum Temperature Data in the Context of Assam", *International Journal of Engineering Sciences & Research Technology*, 4(2), (ISSN : 2277 - 9655), 2015, 255 – 259. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
46. Atwar Rahman & Dhritikesh Chakrabarty, "Elimination of Parameters and Principle of Least Squares: Fitting of Linear Curve to Average Maximum Temperature Data in the Context of Assam", *AryaBhatta J. Math. & Info.* (ISSN (Print): 0975 – 7139, ISSN (Online): 2394 – 9309), 2015, 7(1), 23 – 28. Available at www.abjni.com . Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
47. Atwar Rahman & Dhritikesh Chakrabarty, "Basian-Markovian Principle in Fitting of Linear Curve", The *International Journal Of Engineering And Science,* 2015, 4(6), 31 – 43. Available at www.theijes.com .
50. Atwar Rahman & Dhritikesh Chakrabarty, "Method of Least Squares in Reverse Order: Fitting of Linear Curve to Average Minimum Temperature Data at Guwahati and Tezpur", ,*AryaBhatta J. Math. & Info.* {ISSN (Print): 0975 – 7139, ISSN (Online): 2394 – 9309}, 2015, 7(2), 305 – 312. Available at www.abjni.com . Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
51. Dhritikesh Chakrabarty, "Elimination-Minimization Principle: Fitting of Polynomial Curve to Numerical Data", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 2016, 3(5), 2067 – 2078. Available at www.ijarset.com. Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
52. Dhritikesh Chakrabarty, "Elimination-Minimization Principle: Fitting of Exponential Curve to Numerical Data", *International Journal of Advanced Research in Science, Engineerin and Technology*, (ISSN : 2350 – 0328), 2016, 3(6), 2256 – 2264. *Available at www.ijarset.com.* Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
53. Dhritikesh Chakrabarty, "Impact of Error Contained in Observed Data on Theoretical Model: Study of Some Important Situations", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 2016, 3(1), 1255 – 1265. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
54. Dhritikesh Chakrabarty, "Theoretical Model and Model Satisfied by Observed Data: One Pair of Related Variables", *International Journal of* Advanced Research in Science, Engineering and Technology, 2016 , 3(2), 1527 – 1534, Available in www.ijarset.com . Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats **.**
55. Dhritikesh Chakrabarty, "Variable(s) Connected by Theoretical Model and Model for Respective Observed Data", FSDM2017, *Abstract ID: FSDM2220*, 2017. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.
56. Dhritikesh Chakrabarty, "Numerical Data Containing One Parameter and Random Error: Evaluation of the Parameter by Convergence of Statistic", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2017, 4(2), 59 – 73. Available at *http://eses.net.in/ESES Journal*.
57. Dhritikesh Chakrabarty, "Observed Data Containing One Parameter and Random Error: Evaluation of the Parameter Applying Pythagorean Mean", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2018, 5(1), 32 – 45. Available at *http://eses.net.in/ESES Journal*.
58. Dhritikesh Chakrabarty, "Significance of Change of Rainfall: Confidence Interval of Annual Total Rainfall", *Journal of Chemical, Biological and Physical Sciences* (E- ISSN : 2249 –1929), Sec. C, 2019, 9(3), 151 – 166. Available at: www.jcbsc.org.
59. Dhritikesh Chakrabarty, "Observed Data Containing One Parameter and Random Error: Probabilistic Evaluation of Parameter by Pythagorean Mean", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2019, 6(1), 24 – 40. Available at *http://eses.net.in/ESES Journal*.
60. Dhritikesh Chakrabarty, "Significance of Change in Surface Air Temperature in the Context of India", *Journal of Chemical, Biological and Physical Sciences* (E- ISSN : 2249 – 1929), Sec. C, 2019, 9(4), 251 – 261. Available at: www.jcbsc.org .
61. Dhritikesh Chakrabarty, "Arithmetic-Geometric Mean: Evaluation of Parameter from Observed Data Containing Itself and Random Error", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2019, 6(2), 98 – 111. Available at *http://eses.net.in/ESES Journal*.
62. Dhritikesh Chakrabarty, "AGM: A Technique of Determining the Value of Parameter from Observed Data Containing Itself and Random Error", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 9(3), 2020, 473 – 486. Available at www.jecet.org .

63. Dhritikesh Chakrabarty, "Arithmetic-Harmonic Mean: Evaluation of Parameter from Observed Data Containing Itself and Random Error", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 7(1), 29 – 45, 2020. Available at *http://eses.net.in/ESES Journal*.
64. Dhritikesh Chakrabarty, "Determination of the Value of Parameter μ of the Model $X = \mu + \varepsilon$ by GHM ", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 7(11), 15801 – 15810, 2020. Available at www.ijarset.com.
65. Dhritikesh Chakrabarty, "Central Tendency of Annual Extremum of Surface Air Temperature at Guwahati by AGHM", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 7(12), 16088 – 16098, 2020. Available at www.ijarset.com.
66. Dhritikesh Chakrabarty, "Computational Methods in Analysis of Data: More Measure Central Tendency", NaSAEAST- 2020, Abstract ID: CMAST_NaSAEAST -2002 (I), 2020. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
67. Dhritikesh Chakrabarty, "AGM, AHM, GHM & AGHM: Evaluation of Parameter μ of the Model $X = \mu + \varepsilon$", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 8(2), 16691 – 16699, 2021, Available in www.ijarset.com.
68. *David A. Cox , "The Arithmetic-Geometric Mean of Gauss", In J.L. Berggren; Jonathan M.Borwein; Peter Borwein (eds.).* Pi: A Source Book. *Springer. p. 481.* ISBN 978-0-387-20571-7 *, 2004,* (first published in L'Enseignement Mathématique, t. 30 (1984), p. 275 – 330).
69. Hazewinkel, Michiel, *ed.* , "Arithmetic–geometric mean process, *Encyclopedia of Mathematics*", *Springer Science+Business Media B.V. / Kluwer Academic Publishers, ISBN 978-1-55608-010-4, 2001.*
70. Dhritikesh Chakrabarty, "Comparison of Measures of Parameter of the Model X = μ + ε Based On Pythagorean Means", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN : 2350 – 0328), 8(3), 16948 – 16956, 2021. Available at www.ijarset.com .

AUTHOR'S BIAGRAPHY

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Senior Diploma (in Guitar) from Prayag Sangeet Samiti in 2019 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

(Dr. Dhritikesh Chakrabarty taking mid-day meal with students in Tukurapara Primary School, Kamrup, Assam, in Gunotsav, 2018)

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 8, Issue 5 , May 2021

Dr. Dhritikesh Chakrabarty is also an awardee of the Post Doctoral Research Award by the University Grants Commission for the period 2002─05.

Dr. Dhritikesh Chakrabarty is also an awardee of the Post Doctoral Research Award by the University Grants Commission for the period 2002─05.

He attended five of orientation/refresher course held in Gauhati University, Indian Statistical Institute, University of Calicut and Cochin University of Science & Technology sponsored/organized by University Grants Commission/Indian Academy of Science. He also attended/participated eleven workshops/training programmes of different fields at various institutes.

Dr. Dhritikesh Chakrabarty joined the Department of Statistics of Handique Girls' College, Gauhati University, as a Lecturer on December 09, 1987 and has been serving the institution continuously since then. Currently he is in the position of Associate Professor (& Ex Head) of the same Department of the same College. He had also been serving the National Institute of Pharmaceutical Education & Research (NIPER), Guwahati, as a Guest Faculty continuously from May, 2010 to December,2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years. Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 239 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002─05) and one minor research project (2010─11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Referee of the Journal of Assam Science Society (JASS) and a Member of the Editorial Boards of the two Journals namely (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET) and (2) Journal of Mathematics and System Science. Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.