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Calculation of optimal controls in discrete nonlinear objects with PWM modulation

Ozodov Ezozbek, Yadgarova Dilnoza

Department of Automation and control of technology process, Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Koriy-Niyoziy 39, 100000, Tashkent, Uzbekistan

ABSTRACT: When synthesizing optimal control actions in systems with pulse-width modulation, difficulties arise in obtaining sufficiently simple equations, the solution of which can give correct and acceptable results for further use. In addition, the use of existing methods for solving such problems, even with the use of modern computer technology, cannot guarantee that acceptable results will obtained. The method proposed in the article based on the representation of the dynamics of impulse systems in the form of a space of state variables and the use of the N-interval theorem, which allows the system to transfer to the required state in a minimum number of control cycles.

KEY WORDS: nonlinear signal modulation, interpretation of the dynamics of pulse systems, iterative search for control actions, pulse-width modulation, optimization problem, microprocessor controller.

I. INTRODUCTION

PWM systems are essentially nonlinear systems. The known methods for analyzing the dynamics of this class of systems could classified as methods using various recurrent procedures, methods based on the concept of the phase plane and summary nonlinear equations. Despite the rich and long history of the issue, today there are vast subsets of systems that are either not covered by the known approaches or encounter fundamental difficulties when trying to study them. First, it should be noted such complex subclasses of systems as systems with non-standard modes of operation of latitudinal modulators. In multidimensional systems with autonomous modulators, the repetition periods of impulse elements can be very different. If we take into account that, in this case, the pulse durations at each of the variable intervals for each channel are determined depending on various nonlinear modulation characteristics, then the sources of difficulties in modeling such modes of operation of the pulse parts become clear[2].

The considered PWM circuit consists of pulse modulators and continuous linear part. The duration of the n-th pulse at the output of each of the modulators i = 1,2, ..., N is determined by the value of the error signal e (nT_i) calculated at discrete times, p.s $\tau_n^i = \begin{cases} \varphi^i[e(nT_i)] \ npu \ \varphi^i[e(nT_i)] \le T_i, \\ T \ \ npu \ \ \varphi^i[e(nT_i)] > T_i, \end{cases}$

Where T_i - pulse repetition period at the PWM output

φ^i - Modulation characteristic of a width modulator.

II. METHOD OF RESEARCH

The proposed method is based on a method based on the interpretation of the dynamics of impulsive systems in the form of a space of state variables and the use of the N - interval theorem [4]. The use of this method for systems with nonlinear modulation of the control action requires a modification of the known algorithm for solving the problem of transferring a multidimensional linear dynamic plant with M input and N output controlled variables from a given initial state to the required final state in a minimum number of control cycles. It is assumed that the sampling period is the same for all input signals.

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III. RESULTS OF RESEARCH

The minimum possible number of translation ticks in accordance with the N-interval theorem is determined by the expression : $L = Int\left\{\sum_{i=1}^{N}\sum_{j=1}^{M} P_{ij} / M + 0.5\right\}$ (1)

where P_{ij} - order of the transfer function (differential equation) of the channel j- th entrance; i - th output of the control object[3].

The required state of the control object is determined by the conditions

$$Y_i(L+K) = G_i(L+K), i = \overline{1,N}; K = \overline{0,\tilde{N}_i}, \quad (2)$$

where $Y_i(L+K)$ - the value of the i th output variable in $(L+K)$ - m tick;

$G_i(L+K)$ - required value of the th output variable; G_i - the number of cycles of fixing the th output variable. Based on the analysis of the dynamics of the behavior of the control object, we change conditions (2) to the following:

$$Y_i(L+K) = E_i(L+K), i = \overline{1,N}; K = \overline{0,\tilde{N}_i}, \quad (3)$$

where $E_i(L+K) = G_i(L+K) - Y_i^*(L+K),$ (4)

$Y_i^*(L+K)$ - predicted value of the i-th output variable provided:

$$U_j(m) = 0, j = \overline{1,M}; m = \overline{1,L} \quad (5)$$

Based on dynamic models, it is easy to obtain dependencies connecting the output variables of an object with its input variables for linear impulse systems. In the case of zero initial conditions, these dependencies will have the form:

$$Y_i(L+K) = \sum_{j=1}^{M}\sum_{m=1}^{L} U_j(m) * \omega_{ij}((L+K-m+1)*T), i = \overline{1,N}; K = \overline{0,C_i}, \quad (6)$$

where T - control signal sampling period; $\omega(qT)$ - the value of the weighting function (response to a pulse of duration T) in the th cycle.

Combining the system of expressions (6) with conditions (3), we obtain a system of linear algebraic equations:

$$W * U = E \quad (7)$$

where W - matrix of coefficients of the weighting function:

$$W = \lfloor \omega_{ij}(L+K-m+1) \rfloor; \quad (8)$$

U - vector column of predicted error values:

$$U = [U_1(1), U_2(2), ..., U_1(L), U_2(1), ..., U_2(L), U_m(1), ..., U_m(L)]^T \quad (9)$$

E a column vector of predicted error values:

$$A = [A_1(1), A_1(2), ..., A_1(L), A_2(1), ..., A_2(\tilde{N}_2 + L), A_n(L), ..., E_n(C_i + L)]^T \quad (10)$$

The dimension of system (7) is equal to:

$$M * L = \sum_{i=1}^{N} C_i \quad (11)$$

Having solved system (7), we obtain the desired control actions in the form of linear combinations of predicted errors:

$$U_j(m) = \sum_{i=1}^{N}\sum_{K=0}^{C_1} R_{im}(\sum_{s=1}^{i=1} C_s + K) * E_i(L+K) \quad (12)$$

where R_{im} - vector row matrix ω^{-1}.

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Substituting into expression (12) the values of the predicted errors in accordance with the currently changing initial conditions, we can calculate the numerical values of the control actions.

The obtained expressions (12) are actually the main corrective procedure in the iterative search for control actions odulated in width. But first, let us consider some necessary conditions, the fulfillment of which should ensure the solution of the task. First, the pulse repetition period for pulse width modulation must be equal to the sampling period of the control signal during synthesis for a linear pulse system. Secondly, the condition must be met:

$$|U_j(m)| < A_j \qquad (13)$$

Where A - the amplitude of the width modulated control actions.

If the first condition is not fulfilled it is trivial, then the second one may require significant computational efforts. Let us consider one of the approaches that ensure the fulfillment of condition (13) and consists in an artificial increase in the number of control cycles[4].

Let us assume that the solution of the synthesis problem for a linear impulsive system with the number of translation cycles determined by expression (1) led to the failure of the condition (13). Let's increase the number of translation ticks by J. Let's take it first. Then the values of the predicted errors change, and will be determined by the expression:

$$E_i^j(L+K) = E_i(L+K) - \sum_{j=1}^{M}\sum_{m=1}^{J} U_j(m) * \omega_{ij}(L+K-m+1), \qquad (14)$$

$$i = \overline{1,N}; K = \overline{1; C_i + J}$$

We substitute the found expressions for the predicted errors for expression (12), which allows us to express L basic control actions for each input variable J through additional controls

$$U_j(J+m) = U_j(j+m) + \sum_{K=1}^{M}\sum_{i=1}^{J} \omega_{ij}(L+k-m+1+J) * U_k(i) \qquad (15)$$

Now it is necessary to solve the optimization problem associated with minimizing the criterion:

$$F = \sum_{j=1}^{M}\sum_{i=1}^{L+J} U_j^2(i) \to \min; \ (j = \overline{1,M}; i = \overline{1,J}) \qquad (16)$$

This problem is solved simply by using the least squares method. As a result of its solution, the values of auxiliary control actions are found $U_j(k), (k = 1, J)$. If they all satisfy condition (13), then using formula (14) we find the values of the predicted errors, substitute them into expression (12) and find the values of the control actions $U_j(k), (k = \overline{J+1, L+J})$. They also need to be checked for the fulfillment of condition (13). If it is fulfilled, then you can proceed to the next stage of the synthesis. Otherwise, the value of J must be increased by one and the procedure for minimizing the sum of the squares of the control actions must be repeated[5].

After, as a result of a gradual increase in the number of control cycles, we achieve condition (13), we proceed directly to the iterative procedure for the synthesis of control actions in the class of pulse-width signals. The solution to this problem is based on the proposition that the total area of the control pulses for each output for a linear pulse system and a system with pulse-width control modulation must be equal. In this case, the values of the control actions found during the synthesis for a linear impulse system are corrections in the form of a change in the area of the corresponding control signals modulated in width. Let's consider the main stages of solving the problem:

accept $\tau_i(i) = /U_i(m)/$

$$v(i) = \begin{cases} A_j^* \sin g\{U_j(i)\}, (i-1)*T \prec t \le npu(i=1)*T + \tau_j(i); \\ 0, npu(i-1)*T + \tau_j(i) \prec t \le i*T, \end{cases} \quad \text{where } i = \overline{1,M}; i = \overline{1,J} \qquad (17)$$

These control actions are auxiliary, focused on minimizing the amplitudes of control actions, and therefore, do not change in the future.

1) the state of the system at the moment $t = JT$. This state is taken as the initial one, does not change in the future, and serves to calculate the predicted errors at subsequent points in time.
2) The iteration number is determined

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$q=0; U_j^q(i)=0; \tau_j^q(i)=0;$ where $j=\overline{1,M}; i=\overline{J+1,J+L}$ (18)

3) the values of the transient process in the system are calculated for the moments $t=J+L+K; K=\overline{0,C_i}$.

4) predicted error values are determined $E_i^q(J+L+K); i=\overline{1,N}; K=\overline{0,C_i}$.

5) the formulas (12) are used to calculate the control actions for a linear impulse system. It is suggested that the coefficients R_{jm} defined earlier. Let us denote the found values of the impacts as $U_j(J+K)$.

6) We calculate analogs of control actions for a pulse-width system

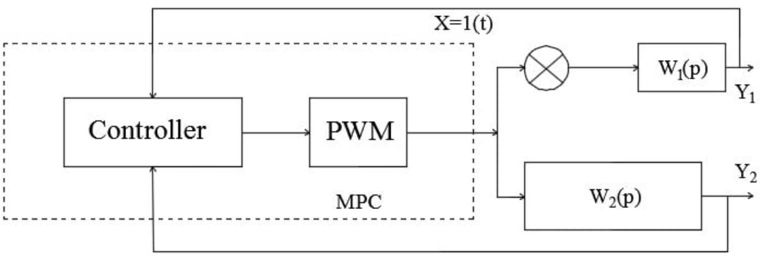
$$U_j^{q+1}(i)=U_j^q(i)+U_j(i) \quad (19)$$

$$\tau_j^{q+1}(i)=/U_j^{q+1}(i)/A_j; j=\overline{1,M}; i=\overline{J+1,J+L} \quad (20)$$

По формуле (18) определяем вид управляющего воздействия.

7) if $/U_j(i)/<\varepsilon; j=\overline{1,M}; i=\overline{J+1,J+L}$ where a predetermined calculation error, therefore, a solution is found and then the transition to step 4 is carried out.

Example: Consider the application of the proposed method by the example of the synthesis of control actions for a system with the structure shown in pic. 1



Pic. 1 Structure of a pulse-width system
Picture 2 here MPC- microprocessor controller, PWM- pulse width modulator.

Let be: $W_1(p)=1/5p+1; W_2(p)=5/2(2p+1); T=1; G_1=5; A=100; G_2=2$

We find: $h_1(t)=L^{-1}[W_2(p)/p]=5(-2+t+2e^{-t/2})$

We calculate the values of transient processes and weight functions:

In order for the steady-state value $Y_1(t)$ it was equal $G_1=5$ accept $X=5$. For this system, we have: $L=3; C_1=0; C_2=1.$ From the coefficients of the weight functions, we form a system of linear equations of the form (7).

$$W=\begin{bmatrix} 0.12152 & 0.14841 & 0.18127 \\ 3.5525 & 2.6135 & 1.0653 \\ 4.122 & 3.5525 & 2.6135 \end{bmatrix}$$

The solution to system (7) gives the following result:

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$$\left.\begin{aligned} U(1) &= -14.3*E_1(3) - 12.07*E_2(3) + 14.88*E_2(4) + 64.8*X \\ U(2) &= -23.07*E_1(3) + 20.254*E_2(3) - 24.2564*E_2(4) - 104.088*X \\ U(3) &= -87.09*E_1(3) - 8.49*E_2(3) + 9.88*E_2(4) + 39.295*X \end{aligned}\right\} \quad (21)$$

Taking the initial conditions as zero, we get:

$E_1(3) = G_1; E_2(3) = G_2; E_2(4) = G_2;$

Substitution of these values obtained into the obtained expressions for $U(k)$ give:

$U(1) = -388.42; U(2) = -625.014; U(3) = -236.186;$

It is easy to verify that condition (13) is not satisfied. We accept $J = 1$,
Then:

$$E_1(4) = G_1 - \omega_1(4)*U(1)$$
$$E_2(4) = G_2 - \omega_2(4)*U(1)$$
$$E_2(5) = G_2 - \omega_2(5)*U(1)$$

Substitution of these expressions in (21) instead of $E_1(3), E_2(3), E_2(4)$ give:

$$U(2) = -388.42 - 2.43*U(1)$$
$$U(3) = 625.014 + 1.93*U(1)$$
$$U(4) = -236.186 - 0.5*U(1)$$

We use the usual least squares procedure to minimize the function:

$$F = \sum_{k=1}^{4} U^2(k) \rightarrow \min$$

$$\frac{\partial F}{\partial U(1)} = 4537.47 + 21.77*U(1) = 0$$

Where do we find $U(1) = -208,44$

Since the found control does not satisfy condition (13), we take $J = J + 1$ and we minimize the function with respect to the variables $U(1)$ and $U(2)$:

$$F = \sum_{k=1}^{5} U^2(k) \rightarrow \min$$

This gives $U(1) = -135,6$ и $U(2) = 21,5$. Again, condition (13) is not met. Therefore, we accept $J = J + 1 = 3$ and minimize the criterion $F = \sum_{k=1}^{6} U^2(k) \rightarrow \min$ by variables $U(1); U(2); U(3)$. As a result, we get:

$U(1) = -86,273; U(2) = -20,73; U(3) = 37,6; U(4) = 73,02; U(5) = 56,77; U(6) = 59,747$

All these values satisfy (13); therefore, one can proceed to an iterative procedure for searching for actions modulated in width.

We accept $\tau_1 = 0,862773; \tau_2 = 0,2073; \tau_3 = 0,3736;$

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$$V_1=\begin{cases}-100;0\le t\le\tau_1\\ 0,\tau_1<t\le 1\end{cases},V_2=\begin{cases}-100;1\le t\le 1.2073\\ 0,1.2073<t\le 2\end{cases},V_3=\begin{cases}-100,2\le t\le 2.3736\\ 0,2..3736<t\le 3\end{cases},$$

Need to find τ_4,τ_5,τ_6 and correspondingly $V4,V5,V6$.

The results of the iterative search in accordance with the described method are shown in the table:

Values of the transition and weight functions

Table 1

t	0	1	2	3	4	5	6	7
$h1(t)$	0	0,1812	0.03296	0.4512	0,5506	0,6312	0,6988	0,7834
$w1(t)$	0	0.1812	0.1418	0.1215	0,0994	0,0814	0,0666	0,0546
$h2(t)$	0	1.0653	3.6768	7.2313	11,353	15,820	20,497	25,302
$w2(t)$	0	1.0653	2.6135	3,5525	4,1220	4,4675	4,6770	4,8041

Values of control actions and output signals

Table 2

g						
0	76.1575	50,48	-56,6	0,7616	0,5048	0,566
1	-29,428	48,97	-19,544	0,4359	1	0,7355
2	7,616	-2,297	-5,3	0,512	0,977	0,79
3	0,03672	-2,834	2,798	0,5124	0,9487	0,79
4	-0,948	1,35	-0,402	-0,402	0,503	0,76
5	0,36	-0,199	-0,16	0,50695	0,9602	0,7647
6	-0,00986	-0,1157	0,1256	0,50645	0,959	0,765
7	-0,051	0,08	-0,029	0,5059	0,96	0,768

The found control actions provide a control error of less than 3%. You can continue iterating if necessary.

IV. CONCLUSION

Quite a lot of works, methods and algorithms are devoted to the solution of the problem of synthesis of optimal control actions in systems with linear modulation of signals [1-4]. The main disadvantages of these methods are their extremely cumbersome and complex mathematical apparatus, a large number of simplifying sentences and calculations, and the complexity of interpreting the results obtained. In addition, the use of these methods often leads to obtaining systems of partial differential equations or algebraic transcendental equations, the exact solution of which is impossible. If some calculations are incorrect, the solution, in principle, may not exist; the use of numerical methods for solving with a large dimension of the resulting system, even when using the capabilities of modern computers, can give an unacceptable result. Therefore, the development of machine-oriented methods for the synthesis of control actions in systems with pulse-width modulation, the application does not require much mathematical training, making intuitive decisions in the implementation of various kinds of assumptions and having a sufficiently large degree of formality, undoubtedly, is an urgent task. One of the approaches to the development of such methods presented in the article.

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