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Asymptotic decomposition of a damped impulse ridden Karman – Donnel equation of an imperfect cylindrical shell

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ABSTRACT: In this work the Karman – Donnel compatibility equations and the Airy's stress function of a damped right circular imperfect cylindrical shell stressed by an impulse load were subjected to multi timing regular perturbation and asymptotic expansions. In doing this, asymptotic series were used to define the outer surface normal deviation W and the Airy stress function F which are all functions of X, Y and T. Consequently, this orchestrated the decomposition of the equations into simpler and solvable partial differential equations in the asymptotic sense. More so, the adoption of the asymptotic series in this transformation reduced wastages of values as very minute, insignificant or negligible values are dropped in the process. Consequently the values obtained after solving the transformed system of equations are more reliable compared to the values obtained after going through the difficulties associated with solving the original system of equations. In a like manner, the adoption of the multi timing regular perturbation technique, in this case, two timing technique is very apt since the original system of equations is involving the small parameters, β measuring the damping parameter and $\overline{W}$ measuring the twice differentiable imperfection parameter. In response to these small parameters, $\overline{W}$ and β , $\in$ and ε were adopted to measure the damping parameter and the twice differentiable imperfection parameter respectively.

KEYWORDS: Impulse, Asymptotic, Perturbation, Cylindrical, Shell

I. INTRODUCTION

Most engineering structures come in the form of cylindrical shells such as beams, machine fittings etc. These cylindrical shells being elastic materials often times undergo several forms of deformation or buckling under different loading conditions. In this work therefore, we are going to look at the decomposition of the Karman – Donnel compatibility equations arising from a damped right circular cylindrical shell struck by an impulse see figure 1.

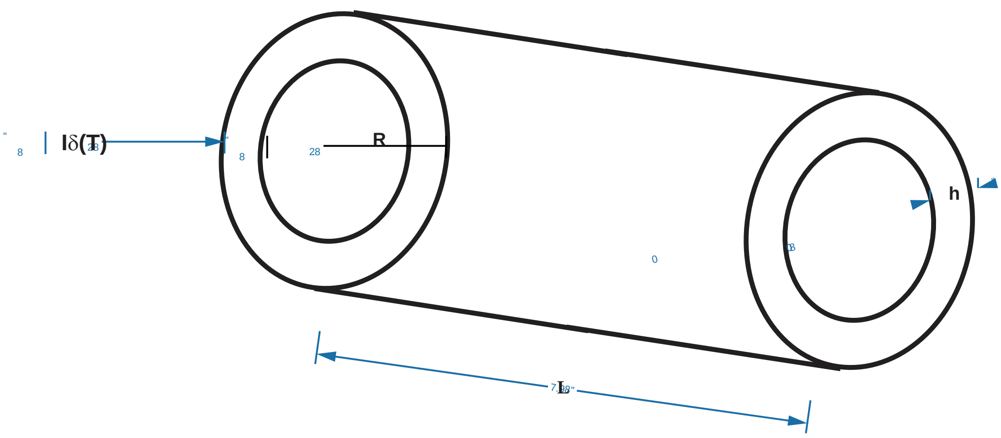

Figure 1: An imperfect cylindrical shell with length L, radius R and thickness h.

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This is an extension of the work of Lockhart and Amazigo [1] who worked on the undamped case with a step load in place. The Karman – Donnel system of equations as obtained is complicated and tasking to mathematicians and those who come across it. Often times analytical solutions appear elusive. However, this research is aimed at finding a better way of solving it. This method is aimed at decomposing the system of equations using asymptotic expansions and regular perturbation, into simpler systems of equations which can be solved separately and aggregated thereafter to obtain the solution of the original problem. Shells generally are regular occurrences in engineering fittings and structures. Several researchers in this area include Ette and Osuji [2], Osuji *et al*. [3], Hu and Burgueno [4], Kriegesmann *et al.* [5] etc. Asymptotic analysis has become a veritable tool in handling small parametric problems as often posed by buckling problems. This is evident in the works of Ette *et al*. [6 - 7] Also the contribution of damping in engineering practice cannot be over-emphasized consequently, several works in this area abound such as Osuji [8], Ette and Osuji [9], Osuji *et al*. [10]. Importantly, the impulse load which is a special type of dynamic loading is a crucial loading phenomenon which can activate a catastrophic dynamic buckling of engineering structures in a split of a second. Owing to its widespread effects, several researchers have over the years delved into its study. These include Ette and Osuji [11], Osuji *et al*. [12]. Given that the cylindrical shell is an imperfect elastic structure, its dynamic stability is subject to prevalent environmental forces. Imperfection in elastic structures is of utmost concern to scientists and engineers, hence, it has attracted the attention of several researchers such as Osuji [13], Paulo *et al*. [14] etc.

II. MATERIALS AND METHOD

The displacement W and Airy's stress function F, satisfy the following equations of motion and compatibility equation (for a damped case).

$$\rho W_{,TT} + D\nabla^4 W + \beta W_{,T} + \frac{1}{R}F_{,XX} = \bar{S}\left(W+\bar{W},F\right) - \Lambda(T) \quad (1)$$

$$\frac{1}{Eh}\nabla^4 F - \frac{1}{R}W_{,XX} = -\bar{S}\left(W,\frac{W}{2}+\bar{W}\right) \quad (2)$$

Where

$$\bar{S}(M,N) = M_{,XX}N_{,YY} + M_{,YY}N_{,XX} - 2M_{,XY}N_{,XY} \quad (3)$$

where $\Lambda(T) = I\delta(T)$ denotes the impulse.

$$0 < X < \pi \quad , \quad 0 < Y < 2\pi \quad , \quad \nabla^4(\ldots) = \left(\frac{\partial^2(\ldots)}{\partial X^2} + \frac{\partial^2(\ldots)}{\partial Y^2}\right)^2$$

$$W = \frac{\partial W}{\partial X} \quad at \quad X = 0,\pi$$

$\bar{W}$ is the twice – differentiable imperfection.

The following are the non-dimensional quantities.

$$x = \frac{X\pi}{L},\; y = \frac{Y}{R},\; \epsilon\bar{w} = \frac{\bar{W}}{h},\; w = \frac{W}{h},\; I\delta(\hat{t}) = \frac{L^2 R\Lambda(t)}{\pi^2 D}$$

$$\varepsilon = \frac{L^2}{\pi^2 R^2},\; A = \frac{L^2\sqrt{12\left(1-\upsilon^2\right)}}{\pi R L},\; K(\xi) = \frac{-h^2}{(1+\xi)^2},\; H = \frac{h}{R}$$

$$0 < \varepsilon << 1\;,\; 0 < \epsilon << 1$$

Let

$$F = \frac{-IR}{2}\left(X^2 + \frac{\alpha Y^2}{2}\right) + \left(\frac{Eh^2 L^2}{\pi^2 R(1+\xi)^2}\right) f \quad (4a)$$

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$$W = \frac{IR^2\left(1-\frac{\alpha\upsilon}{2}\right)}{Eh} + hw \qquad (4b)$$

$$\overline{\nabla}^4(\ldots) = \left(\frac{\partial^2(\ldots)}{\partial x^2} + \xi\frac{\partial^2(\ldots)}{\partial y^2}\right) \text{ is the non – dimensional form of } \nabla .$$

where $\Lambda(T)$ is applied impulse, L denotes height /length of the cylindrical material, while υ represents Poisson's ratio. E denotes the Young's modulus, h represents the fatness of the cylindrical material, R stands for radius, W and F are differentiable functions of X, Y and T while I denotes amplitude of the impulse whose value I_D at buckling we are to determine. β is the small damping coefficient, for $0<\beta<<1$ that is not related to the amplitude of $\overline{W}$. After introducing the non – dimensional quantities in (1) and (2) and simplifying, we obtain

$$\omega_{,\hat{t}\hat{t}} + \overline{\nabla}^4\omega + 2\in\omega_{,\hat{t}} - K(\xi)f_{,xx} + I\delta(\hat{t})\left[\frac{\alpha}{2}(\omega+\varepsilon\overline{\omega})_{,xx} + \xi(\omega+\varepsilon\overline{\omega})_{,yy}\right] = -HK(\xi)S(\omega+\varepsilon\overline{\omega}, f) \qquad (5)$$

$$\overline{\nabla}^4 f - (1+\xi)^2\omega_{,xx} = -H(1+\xi)^2 S\left(\omega, \frac{\omega}{2}+\varepsilon\overline{\omega}\right) \qquad (6)$$

$$0<x<\pi,\ 0<y<2\pi,\ 0<\varepsilon<<1,\ 0<\in<<1 \text{ and } \omega=\omega_{,x}=0 \text{ at } x=0,\ \pi$$

$$\omega(x,y,0^-) = \omega_{,\hat{t}}(x,y,0^-) = 0$$

$$S(M,N) = M_{,xx}N_{,yy} + M_{,yy}N_{,xx} + 2M_{,xy}N_{,xy}$$

Here, $\in$ and ε are not related even though they are small relative to unity. We now integrate (5) from (0^-) to (0^+) where (0^-) is the time at $\hat{t}=0$ just before the action of the impulse while (0^+) is the time at $\hat{t}=0$ after the impulse. We recall that displacement ω and Airy's stress function f are assumed continuous. Thus we have

$$\int_{(0^-)}^{(0^+)}\omega_{,\hat{t}\hat{t}}d\hat{t} + \int_{(0^-)}^{(0^+)}\overline{\nabla}^4\omega d\hat{t} + 2\in\int_{(0^-)}^{(0^+)}\omega_{,\hat{t}}d\hat{t} - \int_{(0^-)}^{(0^+)}K(\xi)f_{,xx}d\hat{t} + I\int_{(0^-)}^{(0^+)}\delta(\hat{t})\left[\frac{\alpha}{2}(\omega+\varepsilon\overline{\omega})_{,xx} + \xi(\omega+\varepsilon\overline{\omega})_{,yy}\right]d\hat{t} = -HK(\xi)\int_{(0^-)}^{(0^+)}[S(\omega+\varepsilon\overline{\omega}, f)]d\hat{t} \qquad (7)$$

Thus

$$\omega_{,\hat{t}}(x,y,0^+) + I\left[\frac{\alpha}{2}\left(\omega(x,y,0^+)+\varepsilon\overline{\omega}(x,y)\right)_{,xx} + \xi(\omega+\varepsilon\overline{\omega})_{,yy}\right] = 0$$

Consequently,

$$\omega_{,\hat{t}}(x,y,0^+) = -I\left[\frac{\alpha}{2}\left(\omega(x,y,0^+)+\varepsilon\overline{\omega}(x,y)\right)_{,xx} + \xi(\omega+\varepsilon\overline{\omega})_{,yy}\right] \qquad (8)$$

Thus after the action of impulse, we get

$$\omega_{,\hat{t}\hat{t}} + 2\in\omega_{,\hat{t}} + \overline{\nabla}^4 f - K(\xi)f_{,xx} = -HK(\xi)S(\omega+\varepsilon\overline{\omega}, f) \qquad (9a)$$

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$$\overline{\nabla}^4 f - (1+\xi)^2 \omega_{,xx} = -H(1+\xi)^2 S\left(\omega, \frac{\omega}{2} + \varepsilon\overline{\omega}\right) \tag{9b}$$

$$0 < x < \pi,\ 0 < y < 2\pi,\ 0 < \varepsilon << 1,\ 0 < \in << 1 \text{ and } \omega = \omega_{,x} = 0 \text{ at } x = 0, \pi$$

$$\omega(x,y,0) = 0,\ \ \omega_{,\hat{t}}(x,y,0) = -I\left[\frac{\alpha}{2}\left(\omega(x,y,0) + \varepsilon\overline{\omega}(x,y)\right)_{,xx} + \xi\left(\omega(x,y,0) + \varepsilon\overline{\omega}\right)_{,yy}\right]$$

Let

$$\tau = \in \hat{t}\quad,\ t = \hat{t} + \left(\frac{\theta_2(\tau)\varepsilon^2 + \theta_3(\tau)\varepsilon^3 + \ldots}{\in}\right) \tag{10}$$

i.e.

$$t = \frac{1}{\in}\left(\theta_2(\tau)\varepsilon^2 + \theta_3(\tau)\varepsilon^3 + \ldots\right) \tag{11}$$

$$\theta_i(0) = 0,\ i = 2,3,4,\ldots$$

Consequently,

$$\omega_{,\hat{t}} = \omega_{,t} + \left(\theta_2'\varepsilon^2 + \theta_3'\varepsilon^3 + \ldots\right)\omega_{,t} + \in \omega_{,\tau} \tag{12a}$$

$$\begin{aligned}\omega_{,\hat{t}\hat{t}} &= \omega_{,tt} + \left(\theta_2'\varepsilon^2 + \theta_3'\varepsilon^3 + \ldots\right)^2 \omega_{,tt} + 2\left(\theta_2'\varepsilon^2 + \theta_3'\varepsilon^3 + \ldots\right)\omega_{,tt} + \in^2 \omega_{,\tau\tau} \\ &\quad + 2\in\left(\theta_2'\varepsilon^2 + \theta_3'\varepsilon^3 + \ldots\right)\omega_{,t\tau} + 2\in\omega_{,t\tau} + \in\left(\theta_2''\varepsilon^2 + \theta_3''\varepsilon^3 + \ldots\right)\omega_{,t}\end{aligned} \tag{12b}$$

Let

$$\omega(x,y,t,\tau) = \sum_{i=1}^{\infty}\sum_{j=0}^{\infty} U^{(i,j)}(x,y,t,\tau)\varepsilon^i \in^j \tag{13a}$$

$$f(x,y,t,\tau) = \sum_{i=1}^{\infty}\sum_{j=0}^{\infty} f^{(i,j)}(x,y,t,\tau)\varepsilon^i \in^j \tag{13b}$$

III. RESULTS

Substituting (13a,b) into (9a,b) we get after simplifying and comparing coefficients of order of ε , $\in$ and their powers such as

$$O(\varepsilon): \begin{cases} U_{,tt}^{(10)} + \overline{\nabla}^4 U^{(10)} - K(\xi) f_{,xx}^{(10)} = 0 \\ \overline{\nabla}^4 f^{(10)} - (1+\xi)^2 U_{,xx}^{(10)} = 0 \end{cases} \tag{14a}$$

$$O(\varepsilon \in): \begin{cases} U_{,tt}^{(11)} + \overline{\nabla}^4 U^{(11)} - K(\xi) f_{,xx}^{(11)} + 2U_{,t}^{(10)} + 2U_{,t\tau}^{(10)} = 0 \\ \overline{\nabla}^4 f^{(11)} - (1+\xi)^2 U_{,xx}^{(11)} = 0 \end{cases} \tag{14b}$$

Etc.

IV. DISCUSSION OF RESULT

The foregoing successfully transformed the earlier complicated system of equations into a comparatively simpler system of equations that can be solved at different stages arising from the orders of the small parameters $\in$ and ε . More so, the adoption of the asymptotic series in this transformation reduced wastages of values as very minute, insignificant or negligible values are dropped in the process. Consequently the values obtained after solving the transformed system of equations are more reliable compared to the values obtained after going through the difficulties associated with solving the original system of equations. In a like manner, the adoption of the multi timing regular perturbation technique, in this case, two timing technique is very apt since the original system of equations is involving the small parameters, β measuring the damping parameter and $\overline{W}$ measuring the twice differentiable imperfection parameter. In response to these small parameters, $\overline{W}$ and β, $\in$ and ε were used to measure the damping parameter and the imperfection parameter respectively. It should be noted that the superscripts (10), (11) etc. do not in any way mean powers but rather orders.

V. CONCLUSION

This investigation has been able to simplify the original difficult system of equations to a series of system of equations which can be solved to any level of order of the small parameters $\in$ and ε. Interestingly, one can choose the level of order at which to stop. But it is worth noting that the higher the order the more the refinement you get in the solutions obtained. The aggregate of these solutions would thereafter give the solution of the original problem. For further investigation, a case of where $\in$ and ε are related should be considered.

VI. STATEMENT OF COMPETING INTEREST

The author has no competing interest.

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