ISSN: 2350-0328
International Journal of AdvancedResearch in Science, Engineering and Technology
Vol. 10, Issue 11, November 2023

$C -$ Almost $P -$ Spaces and Other Spaces on Fuzzy Neutrosophic Topological Spaces

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ABSTRACT: In this paper, we define fuzzy neutrosophic $C -$ almost $P -$ spaces and analyze some new properties of these spaces. Also, we discuss the other spaces of fuzzy neutrosophic topological spaces.

KEYWORDS: Fuzzy neutrosophic regular closed(open) set, Fuzzy neutrosophic $F_\sigma -$ set, Fuzzy neutrosophic $G_\delta -$ set, Fuzzy neutrosophic $C -$ almost $P -$ space, Fuzzy neutrosophic $P -$space, Fuzzy neutrosophic hyper-connected space, Fuzzy neutrosophic extremally disconnected space, Fuzzy neutrosophic basically disconnected space, Fuzzy neutrosophic resolvable space, Fuzzy neutrosophic submaximal space, Fuzzy neutrosophic almost resolvable space.

AMS subject classification: 54A40, 03E72

I. INTRODUCTION

The concept of fuzzy sets was introduced by L.A. Zadeh in 1965 [8]. Then the fuzzy set theory is extension by many researchers. The important concept of fuzzy topological space was offered by C. L. Chang [2] and from that point forward different ideas in topology have been reached out to fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [7] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

In this paper, we define a new concept of fuzzy neutrosophic $C -$ almost $P -$ space and we also discussed some new properties and examples based on this concept. Also, we introduced other spaces of fuzzy neutrosophic topological spaces and examples are investigated.

II. PRELIMINARIES

Throughout the present paper, X denote the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted by $fn(A_N)^+, fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzzy neutrosophic open set $(fnOS)$ if $A_N \leq fn\left(((A_N)^-)^+\right)^-$. The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set $(fnCS)$.

Definition 2.1 [1]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [1]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Definition 2.3 [1]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$
$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

ISSN: 2350-0328
International Journal of AdvancedResearch in Science, Engineering and Technology
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Definition 2.4 [1]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [1]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1\ for\ all\ x\ \in X.$ It is denoted by 0_N.

Definition 2.6 [1]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x\ \in X.$ It is denoted by 1_N.

Definition 2.7 [1]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1]:

A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

(i) $0_N, 1_N \in \tau$

(ii) $A_1 \cap A_2 \in \tau\ for\ any\ A_1, A_2 \in \tau$

(iii) $\cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\} \in \tau$

Satisfying the following axioms.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1]:

Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$int(A) = \cup \{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$

$cl(A) = \cap \{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$

Definition 2.11[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $F_\sigma -$ set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$.

Definition 2.12[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $G_\delta -$ set in (X, τ_N) if $A_N = \wedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 2.13[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic semi-open if $A_N \le fn(\ ((A_N)^+)^-)$. The complement of A_N in (X, τ_N) is called a fuzzy neutrosophic semi-closed set in (X, τ_N).

Definition 2.14[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic dense if there exist no fuzzy neutrosophic closed set B_N in (X, τ_N) such that $A_N \subset B_N \subset 1_X$. That is, $fn\ (A_N)^- = 1_N$.

Definition 2.15[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic nowhere dense set if there exist no non-zero fuzzy neutrosophic open set B_N in (X, τ_N) such that $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Definition 2.16[3]:

Let (X, τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (X, τ_N) is called fuzzy neutrosophic first category set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Any other fuzzy neutrosophic set in (X, τ_N) is said to be of fuzzy neutrosophic second category.

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International Journal of AdvancedResearch in Science, Engineering and Technology
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Definition 2.17[3]:

A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic first category space if the fuzzy neutrosophic set 1_X is a fuzzy neutrosophic first category set in (X, τ_N). That is $1_X = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Otherwise (X, τ_N) will be called a fuzzy neutrosophic second category space.

Definition 2.18[3]:

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $\overline{A_N}$ is called fuzzy neutrosophic residual set in (X, τ_N).

III. Fuzzy Neutrosophic $C -$ almost $P -$ space

Definition 3.1:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic regular – open if $A_N = ((A_N)^-)^+$ and fuzzy neutrosophic regular – closed if $A_N = ((A_N)^+)^-$.

Definition 3.2:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $C -$ almost $P -$ space if for every fuzzy neutrosophic regular- closed set is a fuzzy neutrosophic $G_\delta -$ set A_N in (X, τ_N), $(A_N)^- \neq 0$.

Example3.1:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N *and* C_N defined on X as follows:

A_N: $\{\langle a, 0.5,0.6,0.6\rangle, \langle b, 0.5,0.5,0.5\rangle, \langle c, 0.5,0.6,0.6\rangle\}$

B_N: $\{\langle a, 0.5,0.6,0.5\rangle, \langle b, 0.6,0.6,0.6\rangle, \langle c, 0.5,0.5,0.5\rangle\}$

C_N: $\{\langle a, 0.6,0.5,0.5\rangle, \langle b, 0.4,0.4,0.4\rangle, \langle c, 0.5,0.5,0.6\rangle\}$

Then $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, A_N \wedge C_N, C_N \vee (A_N \wedge B_N), A_N \vee B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

The fuzzy neutrosophic regular closed sets in (X, τ_N) are $1 - B_N, 1 - (A_N \wedge B_N)$, and $1 - (A_N \wedge C_N)$.

Then $1 - B_N, 1 - (A_N \wedge B_N)$, and $1 - (A_N \wedge C_N)$ are fuzzy neutrosophic $G_\delta -$ sets in (X, τ_N).

Thus the fuzzy neutrosophic regular closed sets $1 - B_N, 1 - (A_N \wedge B_N), 1 - (A_N \wedge C_N)$ are fuzzy neutrosophic $G_\delta -$ sets and also $(1 - B_N)^+ \neq 0$, $(1 - A_N \wedge B_N)^+ \neq 0, (1 - A_N \wedge C_N)^+ \neq 0$in (X, τ_N).Hence (X, τ_N) is a fuzzy neutrosophic $C -$ almost $P -$ space.

Definition 3.3:

A fuzzy neutrosophic somewhere dense set if there exists a non - zero fuzzy neutrosophic open B_N in (X, τ_N) such that $B_N < (A_N)^-$.That is $((A_N)^-)^+ \neq 0$ in (X, τ_N).

Proposition 3.1:

If A_N is a fuzzy neutrosophic $F_\sigma -$ set in a fuzzy neutrosophic $C -$ almost $P -$ space (X, τ_N) then $(A_N)^- \neq 1$.

Proof:

Let A_N be a fuzzy neutrosophic $F_\sigma -$ set in a fuzzy neutrosophic $C -$ almost $P -$ space (X, τ_N). Then,$1 - A_N$ is a fuzzy neutrosophic regular closed and fuzzy neutrosophic $G_\delta -$ set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $C -$ almost $P -$ space, for the fuzzy neutrosophic $G_\delta -$ set $(1 - A_N)$, we have $(1 - A_N)^+ \neq 0$. This implies that $1 - (A_N)^- \neq 0$ and hence we have $(A_N)^- \neq 1$.

Proposition 3.2:

If each non-zero fuzzy neutrosophic $G_\delta -$ set is a fuzzy neutrosophic regular closed set in a fuzzy neutrosophic topological space (X, τ_N), then (X, τ_N) is a fuzzyneutrosophic $C -$ almost $P -$ space.

Proof:

Let $A_N$be a non-zerofuzzy neutrosophic $G_\delta -$ set in (X, τ_N) such that $((A_N)^-)^+ = A_N$.We claim that $(A_N)^+ \neq 0$. Assume the contrary. Then $(A_N)^+ \neq 0$ will imply that $((A_N)^-)^+ = (0)^- = 0$ and hence we will have$A_N = 0$, a contradiction to $A_N$beings a non-zero fuzzy neutrosophic $G_\delta -$ set in (X, τ_N).Hence, we must have$(A_N)^+ \neq 0$,for a fuzzy neutrosophic $G_\delta -$ set $A_N$in (X, τ_N) andtherefore(X, τ_N) is a fuzzy neutrosophic $C -$ almost $P -$ space.

Proposition 3.3:

If each non-zero fuzzy neutrosophic $G_\delta -$ set is a fuzzy neutrosophic semi – open set in a fuzzy neutrosophic topological space (X, τ_N), then (X, τ_N) is a fuzzyneutrosophic $C -$ almost $P -$ space.

Proof:

Let $A_N$be a non-zero fuzzy neutrosophic $G_\delta -$ set in (X, τ_N) such that $A_N \leq ((A_N)^+)^-$. We claim that $(A_N)^+ \neq 0$. Assume the contrary we will have $A_N = 0$, a contradiction to A_N beings a non-zero fuzzy

ISSN: 2350-0328
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neutrosophic $G_\delta -$ set in (X,τ_N). Hence, we must have $(A_N)^+ \neq 0$, for a fuzzy neutrosophic $G_\delta -$ set A_N in (X,τ_N) and therefore (X,τ_N) is a fuzzy neutrosophic $C-$ almost$P-$ space.

Proposition 3.4:

If $A_N$is a fuzzy neutrosophic one category set in a fuzzy neutrosophic $C-$ almost $P-$ space in (X,τ_N), then $(A_N)^+ \neq 1$ in (X,τ_N).

Proof:

Let A_N be a fuzzy neutrosophic one category set in (X,τ_N). Then $A_N = \vee_{i=1}^{\infty}(A_{N_i})$, where$(A_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X,τ_N). Now $A_{N_i} \leq \left(A_{N_i}\right)^-$, implies that $\vee_{i=1}^{\infty} A_{N_i} \leq \vee_{i=1}^{\infty}\left(A_{N_i}\right)^-$ and hence we have $[\vee_{i=1}^{\infty}(A_{N_i})]^- \leq [\vee_{i=1}^{\infty}(A_{N_i})^-]^-$. Then $[\vee_{i=1}^{\infty}(A_{N_i})]^- \leq [\vee_{i=1}^{\infty}(A_{N_i})^-]^- \rightarrow (1)$

Now $\vee_{i=1}^{\infty}\left(A_{N_i}\right)^-$ is a fuzzy neutrosophic $F_\sigma -$ set in (X,τ_N). Since (X,τ_N) is a fuzzy neutrosophic $C-$ almost $P-$ space, by proposition 3.1 $[\vee_{i=1}^{\infty}(A_{N_i})^-]^- \neq 1 \rightarrow (2)$.

Now we claim that A_N is not a fuzzy neutrosophic dense set in (X,τ_N). Assume the contrary. Suppose that A_N is a fuzzy neutrosophic nowhere dense set, then $(A_N)^- = 1$, implies from (1), $1 \leq [\vee_{i=1}^{\infty}(A_{N_i})^-]^-$. That is $[\vee_{i=1}^{\infty}(A_{N_i})^-]^- = 1$, a contradiction to (2), hence we must have $(A_N)^- \neq 1$ in (X,τ_N).

Proposition 3.5:

If each non-zero fuzzy neutrosophic one category set is a fuzzy neutrosophic dense set in a fuzzy neutrosophic topological space(X,τ_N), then (X,τ_N) is not a fuzzy neutrosophic $C-$ almost $P-$ space.

Proof:

Let A_N be a fuzzy neutrosophic one category set in (X,τ_N) such that $(A_N)^- = 1$.Then $A_N = \vee_{i=1}^{\infty}(A_{N_i})$, where $(A_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X,τ_N). Now $1-\left(A_{N_i}\right)^-{}'s$ are fuzzy neutrosophic open set in (X,τ_N). Let $B_N = \wedge_{i=1}^{\infty}[1-\left(A_{N_i}\right)^-]$. Then $B_N$fuzzy neutrosophic $G_\delta -$ set in (X,τ_N). Now $B_N = \wedge_{i=1}^{\infty}[1-\left(A_{N_i}\right)^-] = 1-[\vee_{i=1}^{\infty}(A_{N_i})]^- \leq 1-[\vee_{i=1}^{\infty}(A_{N_i})] = 1-A_{N_i}$. That is, $B_N = 1-A_N$. Then $(B_N)^+ \leq (1-A_N)^+$ and hence $(B_N)^+ \leq 1-(A_N)^- = 1-1 = 0_N$. Hence, for the fuzzy neutrosophic $G_\delta -$ set $B_N$in (X,τ_N), $(B_N)^+ = 0$. Therefore (X,τ_N) is not a fuzzyneutrosophic $C-$ almost$P-$ space.

Proposition 3.6:

If $C_N$is a fuzzy neutrosophic residual set in a fuzzy neutrosophic $C-$ almost $P-$ space in (X,τ_N), then $(C_N)^+ \neq 0$ in(X,τ_N).

Proof:

Let C_N be a fuzzy neutrosophic residual set in (X,τ_N). Then $1-C_N$ is a fuzzy neutrosophic first category set in (X,τ_N) and hence by proposition 3.4, $(1-C_N)^- \neq 1$ in (X,τ_N). Therefore $1-(C_N)^+ = (1-C_N)^- \neq 1$. Therefore $(C_N)^+ \neq 0$ in(X,τ_N).

Proposition 3.7:

If $B_N$is a fuzzy neutrosophic residual set in a fuzzy neutrosophic $C-$ almost $P-$ space in (X,τ_N), then B_N is not a fuzzy neutrosophic nowhere dense set in (X,τ_N).

Proof:

Let $B_N$be a fuzzy neutrosophic residual set in a fuzzy neutrosophic$C-$ almost $P-$ space in (X,τ_N), then by proposition 3.6, $(B_N)^+ \neq 0$,in(X,τ_N). We claim that $((B_N)^-)^+ \neq 0$. Assume that contrary. Then $((B_N)^-)^+ = 0$ and $(B_N)^+ \leq ((B_N)^-)^+$, will imply that $(B_N)^+ = 0$, a contradiction. Hence B_N is not a fuzzy neutrosophic nowhere dense set in (X,τ_N).

Proposition 3.8:

If A_N is a fuzzy neutrosophic semi – open setin a fuzzy neutrosophic topological space (X,τ_N), then $(A_N)^-$ is a fuzzy neutrosophic regular closed set in (X,τ_N).

Proof:

If A_N is a fuzzy neutrosophic semi – open set in (X,τ_N) then $A_N \leq ((A_N)^+)^-$ and hence

$$\begin{aligned}(A_N)^- &\leq (((A_N)^+)^-)^- \\ &= ((A_N)^+)^- \\ &\leq [(A_N{}^-)^+]^- \rightarrow (1)\end{aligned}$$

Also, we have $[(A_N)^-]^+ \leq [(A_N)^-]^- = (A_N)^- \rightarrow (2)$

From (1) and (2),

$[((A_N)^-)^+]^- = (A_N)^-$ and hence $(A_N)^-$ is a fuzzy neutrosophic regular closed set in (X,τ_N).

Proposition 3.9:

ISSN: 2350-0328
International Journal of AdvancedResearch in Science, Engineering and Technology
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If A_N is a fuzzy neutrosophic regular open set in a fuzzy neutrosophic $C-$ almost $P-$ space, then A_N is a fuzzy neutrosophic $F_\sigma -$ set in a (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic regular open set in (X, τ_N). Then, $1 - A_N$ is a fuzzy neutrosophic regular closed set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $C-$ almost $P-$space, $1 - A_N$ is a fuzzy neutrosophic $G_\delta -$ set in (X, τ_N). Then, A_N is a fuzzy neutrosophic $F_\sigma -$ set in a (X, τ_N).

Definition

A fuzzy neutrosophic set A_N in fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $\sigma -$ nowhere dense set if A_N is a fuzzy neutrosophic $F_\sigma -$ set in a (X, τ_N) such that $(A_N)^+ = 0$.

Proposition 3.10:

If A_N is a fuzzy neutrosophic regular open set in a fuzzy neutrosophic $C-$ almost $P-$ space, then A_N is not a fuzzy neutrosophic $\sigma -$ nowhere dense set in (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic regular open set in (X, τ_N). Then, $[(A_N)^-]^+ = A_N$, in (X, τ_N) and $[(A_N)^-]^+ = (A_N)^+$. Then $[(A_N)^-]^+ = (A_N)^+$ and $(A_N)^+ = 0$, in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $C-$ almost $P-$space, by proposition 3.1, A_N is a fuzzy neutrosophic $F_\sigma -$ set in a (X, τ_N). Thus A_N is a fuzzy neutrosophic $F_\sigma -$ set with $(A_N)^+ \neq 0$ in (X, τ_N). Hence A_N is not a fuzzy neutrosophic $\sigma -$ nowhere dense set in (X, τ_N).

Proposition 3.11:

If A_N is a fuzzy neutrosophic closed set in a fuzzy neutrosophic $C-$ almost $P-$ space with $(A_N)^+ \neq 0$, then $(A_N)^+$ is a fuzzy neutrosophic $F_\sigma -$ set in a (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic closed set in (X, τ_N) with $(A_N)^+ \neq 0$. Then, $1 - A_N$ is a fuzzy neutrosophic open set in (X, τ_N) such that $(A_N)^- \neq 1$. Since (X, τ_N) is a fuzzy neutrosophic $C-$ almost $P-$space, $(1 - A_N)^-$is a fuzzy neutrosophic $G_\delta -$ set in (X, τ_N) and thus $1 - (A_N)^+$ is a fuzzy neutrosophic $G_\delta -$ set in (X, τ_N). Hence, $(A_N)^+$ is a fuzzy neutrosophic $F_\sigma -$ set in a (X, τ_N).

IV. Other Spaces on Fuzzy Neutrosophic Topological Spaces

Definition 4.1:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $P-$ space if each fuzzy neutrosophic $G_\delta -$ set A_N in (X, τ_N) is fuzzy neutrosophic open set in (X, τ_N).

Example 4.1:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows:

$A_N: \{\langle a, 0.5,0.5,0.6\rangle, \langle b, 0.6,0.6,0.4\rangle, \langle c, 0.6,0.5,0.6\rangle\}$

$B_N: \{\langle a, 0.4,0.5,0.5\rangle, \langle b, 0.5,0.5,0.5\rangle, \langle c, 0.6,0.4,0.5\rangle\}$

$C_N: \{\langle a, 0.6,0.5,0.6\rangle, \langle b, 0.5,0.5,0.6\rangle, \langle c, 0.5,0.4,0.6\rangle\}$

Then $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, A_N \wedge C_N, B_N \wedge C_N, (B_N \wedge C_N) \wedge (A_N \vee B_N), 1_N\}$ is a fuzzy neutrosophic topology on X.

$(A_N)^+ = A_N$, $(B_N)^+ = B_N$, $(C_N)^+ = C_N$, $A_N \vee B_N$, $B_N \vee C_N$, $A_N \vee C_N$, $A_N \wedge B_N$, $B_N \wedge C_N$, $A_N \wedge C_N$, are fuzzy neutrosophic open set in (X, τ_N).

Now the fuzzy neutrosophic sets $B_N \wedge C_N \wedge (A_N \vee B_N) = B_N \wedge C_N$, $(B_N \vee C_N) \wedge (A_N \vee C_N) \wedge (A_N \wedge B_N) = A_N \wedge B_N$ are fuzzy neutrosophic $G_\delta -$ sets in (X, τ_N) shows that fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic $P-$ space.

Definition 4.2:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic hyperconnected space if every non – null fuzzy neutrosophic open subset of (X, τ_N) is fuzzy neutrosophic dense in (X, τ_N).

Example 4.2:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows:

$A_N: \{\langle a, 0.5,0.5,0.6\rangle, \langle b, 0.6,0.5,0.5\rangle, \langle c, 0.6,0.5,0.5\rangle\}$

$B_N: \{\langle a, 0.5,0.6,0.5\rangle, \langle b, 0.7,0.6,0.5\rangle, \langle c, 0.6,0.6,0.4\rangle\}$

$C_N: \{\langle a, 0.4,0.5,0.4\rangle, \langle b, 0.5,0.6,0.5\rangle, \langle c, 0.6,0.4,0.6\rangle\}$

ISSN: 2350-0328
International Journal of AdvancedResearch in Science, Engineering and Technology
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Then $\tau_N = \{0_N, A_N, B_N, C_N,\ A_N \vee B_N,\ A_N \vee C_N,\ B_N \vee C_N,\ A_N \wedge B_N,\ A_N \wedge C_N,\ B_N \wedge C_N,\ A_N \vee B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

$(A_N)^- = 1, (B_N)^- = 1, (C_N)^- = 1,$
$(A_N)^+ = A_N, (B_N)^+ = B_N, (C_N)^+ = C_N,$
$(A_N \vee B_N)^+ = A_N \vee B_N, (B_N \vee C_N)^+ = B_N \vee C_N, (A_N \vee C_N)^+ = A_N \vee C_N,$
$(A_N \wedge B_N)^+ = A_N \wedge B_N, (B_N \wedge C_N)^+ = B_N \wedge C_N, (A_N \wedge C_N)^+ = A_N \wedge C_N,$
$(A_N \vee B_N \vee C_N)^+ = A_N \vee B_N \vee C_N$

If every fuzzy neutrosophic open set is fuzzy neutrosophic dense set. Therefore, fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic hyperconnected space.

Definition 4.3:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic extremally disconnected space if the closure of every fuzzy neutrosophic open set of (X, τ_N) is fuzzy neutrosophic open in (X, τ_N).

Example 4.3:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets $A_N, B_N, C_N, D_N,$ and E_N defined on X as follows:
$A_N: \{\langle a, 0.5,0.4,0.5\rangle, \langle b, 0.3,0.3,0.4\rangle, \langle c, 0.4,0.3,0.4\rangle\}$
$B_N: \{\langle a, 0.5,0.6,0.7\rangle, \langle b, 0.7,0.6,0.5\rangle, \langle c, 0.6,0.5,0.5\rangle\}$
$C_N: \{\langle a, 0.3,0.7,0.3\rangle, \langle b, 0.7,0.3,0.2\rangle, \langle c, 0.2,0.3,0.2\rangle\}$
$D_N: \{\langle a, 0.5,0.6,0.4\rangle, \langle b, 0.7,0.6,0.7\rangle, \langle c, 0.4,0.5,0.4\rangle\}$
$E_N: \{\langle a, 0.3,0.2,0.3\rangle, \langle b, 0.3,0.2,0.2\rangle, \langle c, 0.2,0.3,0.3\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N,\ D_N, E_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

$(A_N)^+ = A_N \Rightarrow (A_N)^- = 1 - B_N = A_N$
$(B_N)^+ = B_N \Rightarrow (B_N)^- = 1 - B_N = A_N$
$(C_N)^+ = C_N \Rightarrow (C_N)^- = 1 - B_N = A_N$
$(D_N)^+ = D_N \Rightarrow (D_N)^- = 1 - B_N = A_N$
$(E_N)^+ = E_N \Rightarrow (E_N)^- = 1 - B_N = A_N$

The closure of every fuzzy neutrosophic open set of (X, τ_N) is fuzzy neutrosophic open set in (X, τ_N).

Definition 4.4:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic basically disconnected space if the closure of every fuzzy neutrosophic open $F_\sigma -$ set of (X, τ_N) is fuzzy neutrosophic open in (X, τ_N).

Example 4.4:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N *and* C_N defined on X as follows:
$A_N: \{\langle a, 0.5,0.5,0.6\rangle, \langle b, 0.7,0.5,0.5\rangle, \langle c, 0.5,0.7,0.6\rangle\}$
$B_N: \{\langle a, 0.5,0.8,0.5\rangle, \langle b, 0.4,0.8,0.5\rangle, \langle c, 0.8,0.4,0.4\rangle\}$
$C_N: \{\langle a, 0.7,0.6,0.7\rangle, \langle b, 0.6,0.5,0.5\rangle, \langle c, 0.5,0.5,0.6\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N,\ A_N \vee B_N,\ A_N \vee C_N,\ B_N \vee C_N,\ A_N \wedge B_N,\ A_N \wedge C_N,\ B_N \wedge C_N,\ 1_N\}$ is a fuzzy neutrosophic topology on X.
Now, the fuzzy neutrosophic $F_\sigma -$ set $D_N = (1 - A_N \wedge B_N) \vee (1 - B_N \wedge C_N) = A_N \vee C_N$.
Implies that $(D_N)^- = 1 - A_N \wedge B_N = A_N \wedge C_N$ is open set in (X, τ_N). Therefore (X, τ_N) is fuzzy neutrosophic basically disconnected space.

Definition 4.5:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic resolvable space if there exists a fuzzy neutrosophic dense set A_N in (X, τ_N) such that $(1 - A_N)^- = 1$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic irresolvable space.

Example 3.5:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N, C_N *and* D_N defined on X as follows:
$A_N: \{\langle a, 0.4,0.7,0.4\rangle, \langle b, 0.7,0.4,0.5\rangle, \langle c, 0.4,0.4,0.4\rangle\}$
$B_N: \{\langle a, 0.6,0.8,0.6\rangle, \langle b, 0.1,0.6,0.1\rangle, \langle c, 0.5,0.1,0.2\rangle\}$
$C_N: \{\langle a, 0.2,0.3,0.8\rangle, \langle b, 0.3,0.1,0.2\rangle, \langle c, 0.1,0.2,0.1\rangle\}$
$D_N: \{\langle a, 0.1,0.7,0.4\rangle, \langle b, 0.7,0.4,0.7\rangle, \langle c, 0.4,0.1,0.1\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N,\ D_N, A_N \vee B_N,\ C_N \vee D_N,\ B_N \vee C_N, A_N \vee D_N,\ A_N \wedge B_N,\ A_N \wedge D_N,\ B_N \wedge C_N,\ C_N \wedge D_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
$(A_N)^- = 1, (1 - A_N)^- = 1,$

ISSN: 2350-0328
International Journal of AdvancedResearch in Science, Engineering and Technology
Vol. 10, Issue 11, November 2023

$(B_N)^- = 1$, $(1 - B_N)^- = 1$
$(C_N)^- = 1$, $(1 - C_N)^- = 1$,
$(D_N)^- = 1$, $(1 - D_N)^- = 1$
This implies that (X, τ_N) is fuzzy neutrosophic resolvable space.

Definition 4.6:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic submaximal space if for each fuzzy neutrosophic set A_N in (X, τ_N) such that $(A_N)^- = 1$ then $A_N \in \tau_N$ in (X, τ_N). That is, (X, τ_N) is a fuzzy neutrosophic submaximal space if each fuzzy neutrosophic dense set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N).

Example 4.6:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows:
A_N: $\{\langle a, 0.5,0.6,0.5\rangle, \langle b, 0.4,0.4,0.4\rangle, \langle c, 0.5,0.5,0.4\rangle\}$
B_N: $\{\langle a, 0.4,0.5,0.4\rangle, \langle b, 0.5,0.4,0.4\rangle, \langle c, 0.5,0.5,0.4\rangle\}$
C_N: $\{\langle a, 0.6,0.6,0.5\rangle, \langle b, 0.5,0.6,0.5\rangle, \langle c, 0.5,0.6,0.4\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, A_N \wedge C_N, B_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
$(C_N)^- = 1$, $(C_N)^+ = C_N$, $(A_N \vee C_N)^- = 1$
$(A_N)^- = 1$, $(A_N)^+ = A_N$.
Therefore (X, τ_N) is fuzzy neutrosophic submaximal space.

Definition 4.7:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic strongly irresolvable space if $((A_N)^-)^+ = 1$, for each fuzzy neutrosophic dense set A_N in (X, τ_N).

Example 4.7:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N and B_N defined on X as follows:
A_N: $\{\langle a, 0.6,0.5,0.6\rangle, \langle b, 0.6,0.6,0.5\rangle, \langle c, 0.5,0.5,0.5\rangle\}$
B_N: $\{\langle a, 0.5,0.7,0.7\rangle, \langle b, 0.7,0.5,0.7\rangle, \langle c, 0.5,0.5,0.5\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \wedge B_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
$(A_N \wedge B_N)^- = 1$, $((A_N \wedge B_N)^-)^+ = 1$.
Now, the fuzzy neutrosophic set $A_N \wedge B_N$ is a fuzzy neutrosophic dense set in (X, τ_N). Implies that $((A_N \wedge B_N)^-)^+ = 1$.Therefore, (X, τ_N) is a fuzzy neutrosophic strongly irresolvable space.

Definition 4.8:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic almost resolvable space if $\bigvee_{i=1}^{\infty}(A_{N_i}) = 1$,where $(A_{N_i})'s$ in (X, τ_N) are such that $\left(A_{N_i}\right)^+ = 0$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic almost irresolvable space.

Example 4.8:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows:
A_N: $\{\langle a, 0.1,0.7,0.3\rangle, \langle b, 0.3,0.7,0.7\rangle, \langle c, 0.7,0.1,0.1\rangle\}$
B_N: $\{\langle a, 0.4,0.1,0.4\rangle, \langle b, 0.1,0.6,0.6\rangle, \langle c, 0.6,0.4,0.1\rangle\}$
C_N: $\{\langle a, 0.5,0.6,0.5\rangle, \langle b, 0.6,0.1,0.1\rangle, \langle c, 0.1,0.5,0.6\rangle\}$
$\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, A_N \wedge C_N, B_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
Then $(A_N)^+ = 0$, $(B_N)^+ = 0$, $(C_N)^+ = 0$ and $\{(A_N) \vee (B_N) \vee (C_N)\} = 1$.
Hence (X, τ_N) is a fuzzy neutrosophic almost resolvable space.

V. CONCLUSION

In this paper, the concept of a new class of spaces called them fuzzy neutrosophic $C -$ almost $P -$spaces and Other spaces on fuzzy neutrosophic topological spaces are defined. Some of its characterizations and examples of fuzzy neutrosophic $C -$ almost $P -$spaces and other spaces on fuzzy neutrosophic topological spaces are also studied. Application to different fields of fuzzy neutrosophic topological spaces such as soft computing, artificial intelligence, decision making, pattern recognition, and image processing. This shall be extended in the future research studies.

ISSN: 2350-0328
International Journal of AdvancedResearch in Science, Engineering and Technology
Vol. 10, Issue 11, November 2023

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