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Additive Property of Harmonic Mean from that of Arithmetic Mean

Dhritikesh Chakrabarty

Independent Researcher, Ex Associate Professor, Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India

ABSTRACT: An additive property of harmonic mean was derived, in a recent study, in the case of discrete variable from its classical definition while the additive property of arithmetic mean is already an established one. Here, the additive property of harmonic mean has been derived from the additive property of arithmetic mean. This derivation of the additive property of harmonic mean, along with numerical example, has been presented in this article.

KEYWORDS: Discrete Variable, Harmonic Mean, Additive Property, Second Proof

I. INTRODUCTION

Average [1 , 37] has been found to be a basic player in playing role of representing /describing characteristic of a set containing many entities by a single one in almost everywhere in academic/research field. Harmonic mean [2 , 5 , 36], one of the three classical means, is a measure of average developed by the great mathematician Pythagoras [4 , 9 , 29 , 32 – 35 , 39] who is the pioneer of developing three measures of average. He defined two other measures of average namely arithmetic mean [2 , 5 , 42] and geometric mean [2 , 5 , 9 , 40 , 43] in addition to harmonic mean These three measures were given the name "Pythagorean Means" [3 , 6] as a mark of honour to him. Later on, a number of definitions / formulations of average had been derived due to necessity of handling different situations. Some of them are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized p mean & generalized p^{th} root mean etc. in addition to arithmetic mean , geometric mean & harmonic mean [7 , 12 , 26 , 27]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [8 , 10 , 11 , 14]. Recently, four formulations of average have been derived from the three Pythagorean means which are Arithmetic-Geometric Mean, Arithmetic-Harmonic Mean, Geometric-Harmonic Mean and Arithmetic-Geometric-Harmonic respectively [13 , 16 – 20].

Measures of average in general and Pythagorean means in particular carry vital importance in developing measure(s) of characteristic(s) of data as well as in developing new and new theoretical concepts/measures [13 – 25 , 41 , 28]. Concepts of arithmetic, geometric and harmonic expectations were introduced on from the concepts of on Pythagorean classical means [28].

Each of the measures of average is to carry its own properties some of whose have already been identified which are available in standard literature of statistics [38]. Of course, there may be many properties which are still to be identified. In the mean time, one property of geometric mean has been identified and termed as its multiplicative property [31]. In another study, one property of harmonic mean has been identified and termed as its additive property [30]. The additive property of harmonic mean was derived, in a recent study, in the case of discrete variable from its classical definition. On the other hand, the additive property of arithmetic mean is already an established one [38]. Here, the additive property of harmonic mean has been derived from the additive property of arithmetic mean. This derivation of the additive property of harmonic mean, along with numerical example, has been presented in this article.

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II. ARITHMETIC AND HARMONIC MEANS

Definition

(1) *Harmonic & Arithmetic Means of a List of Numbers*

Let us consider a list of N real numbers or values namely

$$a_1 , a_2 , \ldots\ldots , a_N$$

Arithmetic mean of $a_1 , a_2 , \ldots\ldots , a_N$, denoted by $A\,(a_1 , a_2 , \ldots\ldots , a_N)$, is defined by

$$A\,(a_1 , a_2 , \ldots\ldots , a_N) = \frac{1}{n}(a_1 + a_2 + \ldots\ldots + a_N)$$

[2 , 5 , 42].

On the other hand, Harmonic Mean of them, denoted by $H\,(a_1 , a_2 , \ldots\ldots , a_N)$, is defined by

$$H\,(a_1 , a_2 , \ldots\ldots , a_N) = \frac{1}{\frac{1}{n}(\frac{1}{a_1} + \frac{1}{a_2} + \ldots\ldots + \frac{1}{a_N})}$$

provided the numbers are non-zero [2 , 5 , 36].

Thus, harmonic mean can be described as the reciprocal of arithmetic mean of the reciprocals.

The definition of harmonic mean implies that

$$H\,(\frac{1}{a_1} , \frac{1}{a_2} , \ldots\ldots , \frac{1}{a_N}) = \frac{1}{\frac{1}{n}(a_1 + a_2 + \ldots\ldots + a_N)}$$

i.e. $$\frac{1}{H(\frac{1}{a_1} , \frac{1}{a_2} , \ldots\ldots , \frac{1}{a_N})} = \frac{1}{n}(a_1 + a_2 + \ldots\ldots + a_N)$$

(2) *Harmonic & Arithmetic Means of a Variable*

If X is a variable which assumes the values

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

Then $A(X)$, the arithmetic mean of X, is defined by

$$A(X) = \frac{1}{n}(x_1 + x_2 + \ldots\ldots + x_n)$$

[2 , 5 , 42].

Also, the harmonic mean of X, denoted by $H(X)$, is defined by

$$H(X) = \frac{1}{\frac{1}{n}(\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots + \frac{1}{x_n})}$$

provided $x_1 , x_2 , \ldots\ldots\ldots , x_n$ are non-zero [2 , 5 , 36].

Definitions of $A(X)$ & $H\,(X)$ imply that

$$H\,(X) = \frac{1}{A(\frac{1}{X})}$$

which implies

$$A\,(X) \;=\; \frac{1}{H(\frac{1}{X})}$$

III. HARMONIC MEAN – ADDITIVE PROPERTY

Additive property of harmonic mean can be stated as follows:

If

$$X_1\,, X_2\,, \ldots\ldots\ldots\ldots\,,\; X_k$$

are k discrete variables such that all assume non-zero values then

$$\frac{1}{H(\frac{1}{X_1+X_2+\ldots\ldots\ldots+\,X_p})} = \frac{1}{H(\frac{1}{X_1})} + \frac{1}{H(\frac{1}{X_2})} + \ldots\ldots\ldots\ldots + \frac{1}{H(\frac{1}{X_p})}$$

Derivation

If X is a discrete variable assuming non-zero values then the arithmetic mean of X is the reciprocal of the harmonic mean of the reciprocal of X i.e. of $\frac{1}{X}$

which implies, $A(X) \;=\; \frac{1}{H(\frac{1}{X})}$

Similarly, if Y is another discrete variable assuming non-zero values the arithmetic mean of Y is the reciprocal of the harmonic mean of the reciprocal of Y i.e. of $\frac{1}{\mathrm{Y}}$

which implies, $A(Y) \;=\; \frac{1}{H(\frac{1}{Y})}$

Accordingly, the arithmetic mean of $(X+Y)$ is the reciprocal of the harmonic mean of the reciprocal of $(X+Y)$ i.e. of $\frac{1}{\mathrm{X+Y}}$

which implies, $A(X+Y) \;=\; \frac{1}{H(\frac{1}{X+Y})}$

But by the additive property of arithmetic mean, the arithmetic mean of $(X+Y)$ is the sum of the individual arithmetic mean of X & Y i.e.

$$A(X+Y) \;= A(X) + A(Y)$$

All these together imply that

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$$\frac{1}{H(\frac{1}{X+Y})} = \frac{1}{H(\frac{1}{X})} + \frac{1}{H(\frac{1}{Y})}$$

Now, suppose that

$$X_1 , X_2 , \ldots\ldots\ldots\ldots , X_p$$

are p discrete variables such each of them assumes non-zero values then by the same logic as in the case of the variable X, then the arithmetic mean of each of them is the reciprocal of the harmonic mean of its reciprocal i.e.

$$A(X_1) = \frac{1}{H(\frac{1}{X_1})} , \quad A(X_2) = \frac{1}{H(\frac{1}{X_2})} , \ldots\ldots\ldots\ldots\ldots , \quad A(X_p) = \frac{1}{H(\frac{1}{X_p})}$$

Similarly, the arithmetic mean of

$$(X_1 + X_2 + \ldots\ldots\ldots + X_p)$$

is the reciprocal of the harmonic mean of the reciprocal of $(X_1 + X_2 + \ldots\ldots\ldots + X_p)$ i.e. of

$$\frac{1}{X_1+X_2+ \ldots\ldots\ldots\ldots + X_p}$$

which implies,

$$A(X_1 + X_2 + \ldots\ldots\ldots\ldots + X_p) = \frac{1}{H(\frac{1}{X_1+X_2+ \ldots\ldots\ldots\ldots + X_p})}$$

But by the generalized additive property of arithmetic mean, the arithmetic mean of the sum of $X_1 , X_2 , \ldots\ldots. , X_p$ is the sum of their individual arithmetic means i.e.

$$A(X_1 + X_2 + \ldots\ldots\ldots\ldots + X_p) = A(X_1) + A(X_2) + \ldots\ldots\ldots\ldots + A(X_p)$$

Therefore,

$$\frac{1}{H(\frac{1}{X_1+X_2+ \ldots\ldots\ldots\ldots + X_p})} = \frac{1}{H(\frac{1}{X_1})} + \frac{1}{H(\frac{1}{X_2})} + \ldots\ldots\ldots\ldots + \frac{1}{H(\frac{1}{X_p})}$$

Corollary:

From the additive property of arithmetic mean it can be obtained that

$$A(c_1X_1 + c_2X_2 + \ldots\ldots\ldots\ldots + c_kX_p) = c_1A(X_1) + c_2A(X_2) + \ldots\ldots\ldots\ldots + c_kA(X_p)$$

for constants $c_1 , c_2 , \ldots\ldots\ldots , c_p ,$

which can be termed as general linear property of arithmetic mean.

Similarly, from the additive property of harmonic mean, it can be obtained that

$$\frac{1}{H(\frac{1}{c_1X_1 + c_2X_2+ \ldots\ldots\ldots\ldots + c_kX_p})} = \frac{c_1}{H(\frac{1}{X_1})} + \frac{c_2}{H(\frac{1}{X_2})} + \ldots\ldots\ldots\ldots + \frac{c_p}{H(\frac{1}{X_p})}$$

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for non-zero constants c_1 , c_2 , ... , c_p ,

which can be termed as general linear property of harmonic mean.

IV. NUMERICAL EXAMPLE

Let X, Y & Z be three variables defined by

X = Integer in the domain [1 , 10] which is a multiple of 3 ,
Y = Integer in the domain [1 , 10] which is a multiple of 5
& Z = Integer in the domain [1 , 10] which is a perfect square

Here X assume the 3 values

3 , 6 , 9 ,

Y assume the 2 values

5 , 10 ,

& Z assume the 3 values

1 , 4 , 9 .

Now,

the variable $X + Y$ assumes the 6 values

8 , 11 , 14 , 13 , 16 , 19,

the variable $Y + Z$ assumes the 6 values

6 , 11 , 9 , 14 , 14 , 19 ,

the variable $Z + X$ assumes the 9 values

4 , 7 , 10 , 7 , 10 , 13 , 12 , 15 , 18 ,

& the variable $X + Y + Z$ assumes the 18 values

9 , 12 , 15 , 12 , 15 , 18 , 17 , 20 , 23 , 14 , 17 , 20 , 17, 20 , 23 , 22 , 25 , 28 .

From computation, it is found that

$$H\left(\frac{1}{X}\right) = 0.16666666666666666666666666666667$$

i.e. $$\frac{1}{H\left(\frac{1}{X}\right)} = 6.0,$$

$$H\left(\frac{1}{Y}\right) = 0.13333333333333333333333333333333$$

i.e. $$\frac{1}{H\left(\frac{1}{Y}\right)} = 7.5,$$

$$H\left(\frac{1}{Z}\right) = 0.21428571428571428571428571428571$$

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i.e. $$\frac{1}{H(\frac{1}{Z})} = 4.66666666666666666666666666666667,$$

$$H\left(\frac{1}{\frac{1}{X+Y}}\right) = 0.0740740740740740740740740740740740$$

i.e. $$\frac{1}{H(\frac{1}{X+Y})} = 13.5,$$

$$H\left(\frac{1}{Y+Z}\right) = 0.0821917808219178082191780821917808219178$$

i.e. $$\frac{1}{H(\frac{1}{Y+Z})} = 12.1666666666666666666666666666667,$$

$$H\left(\frac{1}{Z+X}\right) = 0.08955223880597014925373134328358$$

i.e. $$\frac{1}{H(\frac{1}{Z+X})} = 11.1666666666666666666666666666667,$$

& $$H\left(\frac{1}{X+Y+Z}\right) = 0.05504587155963302752293577981651$$

i.e. $$\frac{1}{H(\frac{1}{X+Y+Z})} = 18.1666666666666666666666666666667.$$

Note that the above numerical results satisfy the following equations:

$$\frac{1}{H(\frac{1}{X})} + \frac{1}{H(\frac{1}{Y})} = \frac{1}{H(\frac{1}{X+Y})},$$

$$\frac{1}{H(\frac{1}{Y})} + \frac{1}{H(\frac{1}{Z})} = \frac{1}{H(\frac{1}{Y+Z})},$$

$$\frac{1}{H(\frac{1}{Z})} + \frac{1}{H(\frac{1}{X})} = \frac{1}{H(\frac{1}{Z+X})},$$

& $$\frac{1}{H(\frac{1}{X})} + \frac{1}{H(\frac{1}{Y})} + \frac{1}{H(\frac{1}{Z})} = \frac{1}{H(\frac{1}{X+Y+Z})}.$$

V. DISCUSSION AND CONCLUSION

The additive property of harmonic mean was derived from its classical definition the in earlier study. In this study, it has been derived from the additive property of arithmetic mean. The aim of this study was to verify whether the two tracks of derivation yield the same result and in the study it has been found so. Consequently, the correctness of the additive property of harmonic mean, as obtained in the earlier study, has also been established by this study.

The additive property of harmonic mean, can be summarized as

$$\frac{1}{\text{Harmonic Mean of }(\frac{1}{\text{Sum of Variables}})} = \text{Sum of }\left\{\frac{1}{\text{Harmonic Mean of }(\frac{1}{\text{Variable}})}\right\}\text{s}$$

This is the rhythm lying in the additive property of harmonic mean. For this reason, additive property of harmonic mean can be regarded as *Rhythmic Additive Property of Harmonic Mean*.

The general linear properties of arithmetic mean, and that of harmonic mean, as outlined in **Corollary**, can be thought of as a basis of developing/deriving more property and/or properties of harmonic mean and also of arithmetic mean.

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AUTHOR'S BIOGRAPHY

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the

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Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

(Dr. Dhritikesh Chakrabarty, in the Department of Statistics of Handique Girls' College, felicitating his former student Dr. Labhita Das for her achievement in higher education of research)

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 260 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS) & (3) Partners Universal International Research Journal (PUIRJ). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.

Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.