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Digital Modeling of Transfer Functions Using State Equations in Controlled Canonical form

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ABSTRACT: In modeling, it is necessary to take into account the influence of the rectifier quantization period. In numerical modeling of an object, the transfer function can be represented by a general rational function. In the control system, the object is given a controlled effect in the form of a step function. Calculating the transfer function of a control system with an analytically known weighting function imposes certain requirements. To obtain a recurrent solution, you need to enter the state space. In the state space, you can obtain controlled canonical forms of the system description. This article considers the issues of numerical modeling of transfer functions using the state equations of the controlled canonical form. Relationships are proposed for use in numerical modeling of transfer functions using the state equations of the controlled canonical form of Frobenius.

KEY WORDS: Model, Object, Transfer Function, Control System, Canonical Form, Quantization Period, Laplace Transform, Weight Function, Coefficient, Polynomials, Frobenius.

I.INTRODUCTION

Any automated control system consists of a set of elements that define its functioning. For each element in the system, the relationship between the input and output variables is described by algebraic, differential, difference equations, or partial differential equations. We refer to discrete systems as those where at least one element corresponds to a discrete change in the output signal, either in time or level, in response to a continuous change in the input signal. Modern computing hardware is equipped with elements featuring a high bit width, allowing us to neglect quantization by level with sufficient accuracy for practical purposes. However, time quantization, although often performed at high frequency (with a small discretization interval), requires an algorithm for a controller generating control actions, which is implemented on a microcontroller through a recurrence relation (difference equation). For the study of digital systems, the z-transform has become widely used. This mathematical tool accounts for time quantization, which is characteristic of digital systems. Level quantization can be disregarded due to the high resolution of modern digital devices [1].

A method for obtaining the state-space description of a two-speed multidimensional discrete control object has been developed in [2], based on the representation of an equivalent discrete system in Jordan canonical form. Additionally, a method for selecting the discrimination periods for the two-speed multidimensional discrete object has been proposed. This method enables the determination of these discrimination periods based on the known state equations of the two-speed multidimensional discrete object and the static characteristics of the analogy-to-digital and digital-to-analogy conversion processes used in the technical implementation of this control task.

Modern instrument manufacturing in the electronics industry is a complex entity that cannot be efficiently coordinated without a digital management system. Implementing such a system allows for the reduction of the development cycle for new products to just a few months and facilitates rapid updates to the product lineup, ensuring high product quality while meeting required production volumes and deadlines. This results in a significant competitive advantage [3].

The dissertation [4] examines the possibility of using state equations in the canonical form of Frobenius for digital modeling of transfer functions. Expressions for the application of state equations in the canonical form of Frobenius for digital modeling of transfer functions are proposed.

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In numerical modeling of a control system, the object model represents the modeled analog object quasi-continuously with a computation step of T_S , while the digital controller is modeled as a discrete algorithm with a quantization period:

$$T_A = l * T_S, \quad l = 1, 2, \ldots \tag{1}$$

When modeling, it is essential to accurately consider the impact of the regulator's quantization period [5-7].

In numerical modeling of an object, the transfer function must be represented as a rational function of general form.

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}, \; n \geq m.$$

The weight function of object $h(t)$ is obtained through the direct application of the inverse Laplace transform:

$$H(s) = G(s) \frac{1}{s} h(t).$$

If $h(t)$ cannot be obtained directly from the correspondence table, which is often the case, calculating the inverse Laplace transform requires the application of the decomposition method. It is necessary to know the location of the poles of the transfer function, specifically the zeros of the denominator polynomial. Denoting these poles as $s_1, s_2, \ldots, s_n$, we can represent $H(s)$ in the case of simple poles $s_i \neq s_j$ as:

$$H(s) = \sum_{k=1}^{n} \frac{c_k}{s - s_k} + \frac{c_0}{s}. \tag{2}$$

Deductions are real or complex constants that are calculated according to a formula:

$$c_k = (s - s_k) H(s) \Big|_{s = s_k}.$$

The case of multiple poles is somewhat more complex and has no practical significance.

By applying the inverse Laplace transform term by term to equation (2), we directly obtain the weight function:

$$h(t) = \sum_{k=1}^{n} c_k e^{s_k t} + c_0.$$

In a control system, the situation is not so straightforward, as the regulated object is influenced by a step function input. Calculating the transfer function of the control system, when the weighting function is known analytically, requires the use of convolution operations [8,9].

This involves computing the convolution integral:

$$y(t) = \int_0^t g(v) u(t - v) dv = \int_0^t u(t - v) dh$$

It requires knowledge, on one hand, of the weighting function $g(t) = dh / dt$ or the transfer function $h(t)$ of the object and, on the other hand, of the input signal $u(t)$, which represents a sequence of control action values.

Since the quantization period T_A is a multiple of the computation step T_S, it makes sense to approximate the convolution integral with a sum of the form.

$$y(iT_S) = \sum_{j=0}^{i} T_S g(jT_S) u((i - j)T_S).$$

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Assuming $y_i := y(iT_S)$, let's rewrite this relationship as:

$$y_i = T_S\left[u_i,\ldots,u_0\right]\begin{bmatrix} g_0 \\ . \\ . \\ . \\ g_i \end{bmatrix}.$$

This equality holds not only in the case of $T_S = T_A$ but also in the general case (1), since the input value u_j remains constant at moments $l * T_S$:

$$u_j = u_{j+1} = \ldots = u_{j+1-1} .$$

An obvious drawback of the convolution operation is that it does not allow for representation in a recurrent form. To obtain a recurrent solution, it is necessary to transition to state space. For the system description in state space, we will use the so-called controlled canonical form, where the system matrix takes the form of a Frobenius matrix.

The elements of the matrices in this representation are derived directly from the coefficients of the polynomials involved in $G(s)$, without any computations:

$$\dot{x} = Ax + bu, \quad y = c^T x, \tag{3}$$

where:

$$A = \begin{bmatrix} 0 & -1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1 \\ -a_0 & -a_1 & -a_2 & . & . & . & -a_{n-1} \end{bmatrix}, b\begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{bmatrix},$$

$$c^T = \left[b_0 b_1 \ldots b_m 0 \ldots . 0\right]. \tag{4}$$

To obtain the state vector x at regular time intervals during which the input value u changes abruptly, we have the recurrence relation:

$$x_{i+1} = \exp(AT_S)x_i + (\exp(AT_S) - I)A^{-1}bu_i , \tag{5}$$

where:

$$i = 0,1,2,\ldots,$$
$$T_S = t_{i+1} - t_i ,$$
$$x_i = x(t_i),$$
$$u_i = u(t) = const.\ fur\, t_i \le t \le t_{i+1}.$$

The relation (5) is derived under the practically insignificant assumption that the object has no poles at $s = 0$.

It should be noted that relation (5) provides exact values, rather than approximate ones, at quantization moments.

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When the equation of state does not have a specific structure, the use of the transition matrix in modeling encounters the following operations that can cause numerical difficulties:

1. Matrix inversion A;
2. Calculation of the transition matrix $\exp(AT_S)$;
3. Calculation of the multiplicative term for u_i in the form of $(\exp(AT_S)-I)*A^{-1}b$.

All three tasks are significantly simplified for the controlled canonical form.

It is proved that the matrix A of the type (4) has the following inverse matrix:

$$A^{-1}=\begin{bmatrix} -a_1/a_0 & -a_2/a_0 & . & . & . & -a_{n-1}/a_0 & -1/a_0 \\ 1 & 0 & . & . & . & . & 0 \\ 0 & 1 & . & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & . & . & . & . & 1 & 0 \end{bmatrix}.$$

The transition matrix $\exp(AT_S)$ is defined as an infinite series:

$$\exp(AT_S)=I+AT_S/1!+A^2T_S^2/2!+\ldots \tag{6}$$

it is usually calculated using a finite number of terms from this series. The powers of matrix A in the case of the Frobenius form are computed relatively simply.

Let us denote the elements of matrix A^{k-1} as d_{ij}, and the elements of matrix A^k as e_{ij}. Then:

$$e_{ij}=d_{i+1,j} \quad fur \;\; i=1,2,\ldots,n-1$$
$$und \;\; j=1,2,\ldots,n,$$
$$e_{n1}=-d_{nn}a_0,$$
$$e_{nj}=d_{n,j-1}-d_{nn}a_{j-1} \quad fur \;\; j=2,3,\ldots,n.$$

To calculate the exponential series (6), we will introduce the following notations:

$$\exp(AT_S):=E=E^{(1)}+E^{(2)}+E^{(3)}+\ldots,$$

where:

$$\mathrm{E}^{(0)}:=I.$$

Then for:

$$k=1,2,3,\ldots:$$
$$E_{i,j}^{(k)}=E_{i+1,j}^{(k-1)}T_S/k, \quad i=1,2,\ldots,n-1.$$
$$j=1,2,\ldots,n,$$
$$E_{n,j}^{(k)}=(E_{n,j-1}^{(k-1)})-E_{n,n}^{(k-1)}a_{j-1}T_S/k,$$
$$j=2,\ldots,n,$$
$$E_{n,n}^{(k)}=-E_{n,n}^{(k-1)}a_0T_S/k.$$

Note that when calculating the transition matrix, it may be necessary to consider a large number of series terms, as matrix AT_s may be ill-conditioned (have a large norm). In this case, we will transform the expression for the transition matrix as follows:

$$\exp(AT_S) = (\exp(AT_S / q))^q .$$

After this, we will calculate the value of the exponential series for matrix AT_S / q with the required precision. Choosing:

$$q = 2^p ,$$

the resulting value must be squared p times.

For the multiplicative term associated with u_i, we have the following simple expression:

$$(\exp(AT_S) - I)A^{-1}b = \frac{-1}{a_0}\begin{bmatrix} E_{11} - 1 \\ E_{21} \\ . \\ . \\ . \\ E_{n,1} \end{bmatrix}.$$

And finally, the desired value of the output signal is obtained according to equations (3) using the formula:

$$y_k = c^T x_k .$$

Thus, the given relationships can be used in digital modeling of transfer functions using equations of state in the controlled canonical Frobenius form.

II.CONCLUSION

To obtain a recurrent solution, you need to enter the state space. In the state space, you can obtain controlled canonical forms of the system description. Relationships are proposed for use in numerical modeling of transfer functions using the state equations of the controlled canonical form of Frobenius.

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