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Average: A Basis of Measures of Dispersion of Data

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ABSTRACT: Data in general and numerical data in particular carry various characteristics which are to be measured. It has been found that average is a basis of mathematical measures of central tendency of data. Here attempt has been made on establishing the fact that average is also a basis of mathematical measures of dispersion of data. The derivation/justification of average as a basis of measures of dispersion of data has been presented in this article.

KEYWORDS*:* Average, data, dispersion, basis of measures

I. INTRODUCTION

Average [2 , 20 , 52], a commonly speaking term, is a concept which refers to an aggregate but not to individual while averaging is a popular technique of describing/representing a significance/ characteristic of an aggregate/population/class of individuals overall by a single entity. An average is a single number taken as representative of a non-empty list of numbers (including a large set of numbers). He is Pythagoras [3 , 48 , 55] who is the pioneer of defining average and formulating its measure. He defined the three most common averages namely arithmetic mean, geometric mean and harmonic mean which were given the name "Pythagorean Means" [4 , 6 , 7 , 19] as a mark of honour to him. Later on, a number of definitions / formulations of average had been derived due to necessity of handling different situations. Some of them are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized p *m*ean & generalized p^{th} root mean etc. in addition to Arithmetic Mean , Geometric Mean & Harmonic Mean [8 , 10, 51]. The next attempt was towards the development of generalized definitions/formulations of average. In this respect, three generalized definitions of average had been developed which are Generalized f - Mean [10 , 11*]*, Generalized f_H – Mean [12] & Generalized f_G – Mean [13 , 15]. In another studies, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining averag e of a function of a set (or of a list) of values [13 , 15 , 16 , 23]. Recently, four formulations of average have been derived from the three Pythagorean means which are Arithmetic-Geometric Mean, Arithmetic-Harmonic Mean, Geometric-Harmonic Mean and Arithmetic-Geometric-Harmonic respectively [5 , 31 , 32 , 49 , 50].

However, data in general and numerical data in particular carry various characteristics like central tendency, dispersion etc. [1 , 3 , 9 , 33 , 53 , 54 , 56 , 57] which are to be measured. The three Pythagorean means along with the four formulations derived from them are mathematical measures of central tendency of data. [1 , 21 − 30 , 33 − 47 , 53 , 57] which implies that average is a basis of mathematical measures of central tendency of data. Here attempt has been made on establishing that average is also a basis of mathematical measures of dispersion of data. The derive ation/justification of average as a basis of measures of dispersion of data has been presented in this article.

II. PYTHAGOREAN MEANS AND MEASURES OF AVERAGE OBTAINED FROM THEM

Let us consider a list of n numbers or values namely

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$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

Arithmetic Mean of them is calculated by

$$\frac{1}{n}(x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots + x_n)$$

Geometric Mean of them is calculated by

$$(x_1 , x_2 , \ldots\ldots , x_n)^{1/n}$$

or equivalently by

$$antilog\ \{\frac{1}{n}(log\ x_1 + log\ x_2 + \ldots\ldots\ldots + log\ x_n)\}$$

provided the *n* numbers are strictly positive.

Harmonic Mean of them is calculated by

$$\frac{1}{\frac{1}{n}(\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots\ldots + \frac{1}{x_n})}$$

provided the *n* numbers are non-zero.

Arithmetic-Geometric Mean, Arithmetic-Harmonic Mean, Geometric-Harmonic Mean & Arithmetic-Geometric-Harmonic Mean are the four formulations of average that have been derived from the three Pythagorean means.

Let a_0 , g_0 & h_0 be respectively the Arithmetic Mean, the Geometric Mean & the Harmonic Mean of the *n* numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

Then Arithmetic-Geometric Mean, Arithmetic-Harmonic Mean, Geometric-Harmonic Mean & Arithmetic-Geometric-Harmonic Mean of them are defined as follows:

Arithmetic-Geometric Mean of $x_1 , x_2 , \ldots\ldots\ldots , x_n$ is the common point (value) of convergence of the two sequences

$$\{ a_n \}\ \&\ \{ g_n \}$$

defined by

$$a_{n+1} = \tfrac{1}{2}(a_n + g_n)$$
$$\&\quad g_{n+1} = (a_n\, g_n)^{1/2}$$

respectively where the square root takes the principal value .

Arithmetic--Harmonic Mean of $x_1 , x_2 , \ldots\ldots\ldots , x_n$ is the common point (value) of convergence of the two the two sequences $\{a'_n\}$ & $\{ h'_n \}$ defined respectively by

$$a'_{n+1} = \tfrac{1}{2}(a'_n + h'_n) \quad \&\quad h'_{n+1} = \tfrac{1}{2}(a'^{-1}_n + h'^{-1}_n)\}^{-1}$$

where $a'_0 = a_0$, $g'_0 = g_0$ & $h'_n = h_0$.

Geometric-Harmonic Mean of $x_1 , x_2 , \ldots\ldots\ldots , x_n$ is the common point (value) of convergence of the two sequences $\{g''_n\}$ & $\{h''_n\}$ defined respectively by

$$g''_{n+1} = (g''_n \,.\, h''_n)^{\frac{1}{2}}$$
$$\&\quad h''_{n+1} = \{\tfrac{1}{2}(g''^{-1}_n + h''^{-1}_n)\}^{-1}$$

with $a''_0 = a_0$, $g''_0 = g_0$ & $h''_n = h_0$

where the square cube takes the principal value.

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Arithmetic-Geometric-Harmonic Mean of $x_1 , x_2 , \ldots\ldots\ldots , x_n$ is the common point (value) of convergence of the
three sequences $\{a'''_n\}$, $\{ g'''_n \}$ & $\{h'''_n \}$ defined respectively by

$$a'''_{n+1} = 1/3\,(a'''_n + g'''_n + h'''_n) \ ,$$
$$g'''_{n+1} = (a'''_n\, g'''_n\, h'''_n)^{1/3}$$
$$\& \quad h'''_{n+1} = \{1/3\,(a'''_n{}^{-1} + g'''_n{}^{-1} + h'''_n{}^{-1})\}^{-1}$$

with $a'''_0 = a_0$, $g'''_0 = g_0$ & $h'''_n = \mathrm{h}_0$
where the cube root takes the principal value.

III. AVERAGE AS A BASIS OF MEASURES OF DISPERSION

In order to examine the role of average in measures of dispersion of data let us first consider the average of a function of variable.

Let $\xi(x)$ be a function of x.

Then $\xi(x)$ assumes the values

$$\xi(x_1) , \xi(x_2) , \ldots\ldots\ldots\ldots , \xi(x_n)$$

corresponding to the values $x_1 , x_2 , \ldots\ldots\ldots , x_n$ assumed by x.

In this case, arithmetic Mean of the function $\xi(x)$ is given by

$$\frac{1}{n}\sum_{i=1}^{n} \xi(x_i)$$

Similarly, geometric Mean of the function $\xi(x)$ is given by

$$[\prod_{i=1}^{n} \ \xi(x_i)]^{1/n}$$

Also, harmonic Mean of the function $\xi(x)$ is given by

$$\frac{1}{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{\xi(x_i)}}$$

Raw Moment (about origin)

Suppose, the arithmetic mean of X i.e. of the values

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

is α.

Let us put $\xi(x) = \mathrm{x}^{\mathrm{r}}$
Then the arithmetic mean of $\xi(x)$ is given by

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

This is nothing but the raw moment of X of order r about origin.
Thus, the raw moment of X of order r about origin can be defined as the arithmetic mean of X^r.

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Factorial Raw Moment (about origin)

Let us put $\xi(x) = x^{(r)}$

Then the arithmetic mean of $\xi(x)$ is given by

$$\frac{1}{n}\sum_{i=1}^{n} x_i^{(r)}$$

This is nothing but the raw factorial moment of X of order r about origin.

Thus, the raw factorial moment of X of order r about origin can be defined as the arithmetic mean of $\mathrm{X}^{(\mathrm{r})}$.

Raw Moment (about arbitrary point 'a')

Let us put $\xi(x) = (x-a)^r$

Then the arithmetic mean of $\xi(x)$ becomes

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-a)^r$$

This is nothing but the raw moment of X of order *r* about the arbitrary point '*a*'.

Thus, the raw moment of X of order r about an arbitrary point 'a' can be defined as the arithmetic mean of $(X-a)^r$.

Raw Factorial Moment (about arbitrary point 'a')

Let us put $\xi(x) = (x-a)^{(r)}$

Then the arithmetic mean of $\xi(x)$ becomes

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-a)^{(r)}$$

This is nothing but the raw factorial moment of X of order r about arbitrary point 'a'.

Thus, the raw factorial moment of X of order r about arbitrary point 'a' can be defined as the arithmetic mean of $(X-a)^r$.

Central Moment

Let us put $\xi(\mathrm{x}) = (x-\alpha)^r$

Then the arithmetic mean of $\xi(x)$ becomes

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-\alpha)^r$$

This is nothing but the central moment of X of order r.

Thus, the central moment of X of order *r* can be defined as the arithmetic mean of $(X-\alpha)^r$.

Note (3.1):

In particular, if $r = 2$

then it becomes

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-\alpha)^2$$

This is nothing but the variance of X.

Thus, the variance of X can be defined as the arithmetic mean of $(X-\alpha)^2$.

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Central Factorial Moment

Let us put $\xi(x) = (x-\alpha)^{(r)}$

Then the arithmetic mean of $\xi(x)$ becomes

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-\alpha)^{(r)}$$

This is nothing but the central factorial moment of X of order r.

Thus, the central factorial moment of X of order r can be defined as the arithmetic mean of $(X-\alpha)^{(r)}$.

Gini's measure of variation

If we put

$$\xi(x_i) = (x_i - x_j)^2 \quad , \quad i < j \ ,$$

then the arithmetic mean of $\xi(x_i)$ becomes

$$\frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=2\\ i<j}}^{n}(x_i-x_j)^2$$

This is nothing but Gini's measure of variation of X .

Thus, Gini's measure of variation of X can be defined as the arithmetic mean of $(x_i - x_j)^2$.

Co- variation

Suppose, Y is another variable which assumes the values

$$y_1 , y_2 , \ldots\ldots\ldots , y_n$$

whose arithmetic mean is β.

If we put $\xi(x , y) = (x-\alpha)(y-\beta)$,

then the arithmetic mean of $\xi(x , y)$ becomes

$$\frac{1}{n}\sum_{i=1}^{n}\sum_{j}^{n}(x_i-\alpha)(y_i-\beta)$$

This is nothing but the covariance between X & Y .

Thus, covariance between X & Y can be defined as the arithmetic mean of $(X-\alpha)(Y-\beta)$.

IV. CONCLUDING REMARK

Average is the basis of the mathematical measures of dispersion computed from data. Each of the mathematical measures of dispersion of a variable based on data can be interpreted as an average of some function of the associated variable.

Finally, let us conclude with the hypothesis that average is the basis of the mathematical measures of that which is to be computed from data.

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AUTHOR'S BIOGRAPHY

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of

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the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

(Dr. Dhritikesh Chakrabarty, at the left in front line, in Concordia University, Montreal, Quebec, Canada, while participating in the International Conference "Statistics 2011 Canada / IMST 2011– FIM XX" during July 1 – 4, 2011)

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 260 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR),

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(3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS) & (3) Partners Universal International Research Journal (PUIRJ). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.

Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.

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