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Additive Property of Harmonic Mean

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ABSTRACT: Harmonic mean, one of the three classical means developed by Pythagoras, has wide application in many situations. The properties of harmonic mean, which are yet unknown or unestablished, are to be known or to be established due to its importance in reality. In this study, additive property of harmonic mean has been derived. Derivation of the property has been presented in this article along with numerical example.

KEYWORDS: Harmonic Mean, Additive Property, Derivation

I. INTRODUCTION

The concept of average [1, 32] found to be used in almost everywhere. Measure of average was first developed by the great mathematician Pythagoras [4, 28, 29, 30, 31, 34]. He defined three measures of average namely arithmetic mean [2, 5, 21, 26], geometric mean [2, 5, 9, 21, 26] and harmonic mean [2, 5, 21, 26, 36] which were given the name "Pythagorean Means" [3, 6, 9, 27, 33, 35] as a mark of honour to him. Later on, a number of definitions / formulations of average had been derived due to necessity of handling different situations. Some of them are quadratic mean or root mean square, square root mean, cubic mean, cube root mean, generalized p mean & generalized p^{th} root mean etc. in addition to Arithmetic Mean, Geometric Mean & Harmonic Mean [7, 10, 12, 26]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [8, 10, 11, 14]. Recently, four formulations of average have been derived from the three Pythagorean means which are Arithmetic-Geometric Mean, Arithmetic-Harmonic Mean, Geometric-Harmonic Mean and Arithmetic-Geometric-Harmonic respectively [13, 15-20, 22-26].

Each of the measures of average is to carry its own properties of whose some are known. It is important to know whether a measure carry additive property since it can be useful for many tasks. Harmonic mean, one of the three classical means developed by Pythagoras, has wide application in many situations. The properties of harmonic mean, which are yet unknown or unestablished, are to be known or to be established due to its importance in reality. In this study, additive property of harmonic mean has been derived. Derivation of the property has been presented in this article along with numerical example.

II. HARMONIC MEAN

Let us consider a list of *n* real numbers or values namely

$$x_1$$
 , x_2 , ,

 x_1, x_2, \dots, x_n Harmonic Mean [[2, 5, 21, 26, 36], denoted by $H(x_1, x_2, \dots, x_n)$, of them is defined by

$$H(x_1, x_2, \dots, x_n) = \frac{1}{\frac{1}{n}(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n})}$$

provided the numbers are non-zero.

On the other hand, arithmetic mean [2, 5, 21, 26] of x_1 , x_2 , ..., x_n , denoted by $A(x_1, x_2, \dots, x_n)$, is defined by

$$A(x_1, x_2, \dots, x_n) = \frac{1}{n}(x_1 + x_2 + \dots + x_n)$$

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Thus harmonic mean can be described as the reciprocal of arithmetic mean of the reciprocals.

The definition of harmonic mean implies that

$$H(\frac{1}{x_1}, \frac{1}{x_2}, \dots, \frac{1}{x_n}) = \frac{1}{\frac{1}{n}(x_1 + x_2 + \dots + x_n)}$$

i.e.
$$\frac{1}{H(\frac{1}{x_1}, \frac{1}{x_2}, \dots, \frac{1}{x_n})} = \frac{1}{n}(x_1 + x_2 + \dots + x_n)$$

III. ADDITIVE PROPERTY OF HARMONIC MEAN

If X is a variable which assumes the values

$$x_1, x_2, \dots, x_n$$

then by definition,

harmonic mean of X, denoted by H(X), will be

$$H(X) = \frac{1}{\frac{1}{n}(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n})}$$

which implies

$$\frac{1}{H(\frac{1}{X})} = \frac{1}{n}(x_1 + x_2 + \dots + x_n)$$

Similarly, if another variable Y which assumes the non-zero values

$$y_1, y_2, \dots, y_m$$

then

$$H(Y) = \frac{1}{\frac{1}{m}(\frac{1}{y_1} + \frac{1}{y_2} + \dots + \frac{1}{y_m})}$$

which implies,

$$\frac{1}{H(\frac{1}{Y})} = \frac{1}{m}(y_1 + y_2 + \dots + y_m)$$

Now the variable X + Y assumes the n + m values

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Accordingly,

$$\frac{1}{H(\frac{1}{X+Y})} = \frac{1}{n+m} \{ (x_1 + y_1) + (x_1 + y_2) + \dots + (x_1 + y_m) + (x_1 + y_1) + (x_1 + y_2) + \dots + (x_1 + y_m) + (x_1 + y_m) + (x_1 + y_m) \}$$

$$= \frac{1}{n} (x_1 + x_2 + \dots + x_n) + \frac{1}{m} (y_1 + y_2 + \dots + y_m)$$

i.e.
$$\frac{1}{H(\frac{1}{X+Y})} = \frac{1}{H(\frac{1}{X})} + \frac{1}{H(\frac{1}{Y})}$$

Now, if X, Y & Z are three discrete random variables such that all assume non-zero values then

$$X + Y + Z = X + (Y + Z)$$

where (Y + Z) is a variable.

Accordingly,

$$\frac{1}{H(\frac{1}{X+Y+Z})} = \frac{1}{H(\frac{1}{X+Y})} + \frac{1}{H(\frac{1}{Z})}$$

i.e.
$$\frac{1}{H(\frac{1}{X+Y+Z})} = \frac{1}{H(\frac{1}{X})} + \frac{1}{H(\frac{1}{Y})} + \frac{1}{H(\frac{1}{Z})}$$

By the application the mathematical induction, one can obtain that if

 X_1, X_2, \dots, X_p

are p discrete random variables such that all assume non-zero values then

$$\frac{1}{H(\frac{1}{X_1+X_2+\dots+X_p})} = \frac{1}{H(\frac{1}{X_1})} + \frac{1}{H(\frac{1}{X_2})} + \dots + \frac{1}{H(\frac{1}{X_p})}$$

Thus the following *theorem*, interpretable as additive property of harmonic expectation, has been obtained:

Theorem (3.1):

The reciprocal of harmonic mean of the sum of a number of variables is equal to the sum of the reciprocals of the individual harmonic means of the reciprocals of the respective variables i.e. if

$$X_1$$
, X_2 ,, X_p

are p discrete random variables such that all assume non-zero values then

$$\frac{1}{H(\frac{1}{X_1 + X_2 + \dots + X_p})} = \frac{1}{H(\frac{1}{X_1})} + \frac{1}{H(\frac{1}{X_2})} + \dots + \frac{1}{H(\frac{1}{X_p})}$$

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IV. NUMERICAL EXAMPLE

If X is a variable which denotes odd integer lying between 1 & 10 then it assumes the values

After calculation, it is found that

$$H(\frac{1}{X}) = 0.2$$
 i.e. $\frac{1}{H(\frac{1}{X})} = 5$

Similarly if Y is another variable which denotes even integer lying between 1 & 10 then it assumes the values

After calculation, it is found that

The variable X + Y assumes the 25 values

3, 5, 7, 9, 11, 5, 7, 9, 11, 13, 7, 9, 11, 13, 15, 9, 11, 13, 15, 17, 11, 13, 15, 17, 19 After calculation, it is found that

Note that

$$\frac{1}{H(\frac{1}{X})} + \frac{1}{H(\frac{1}{Y})} = 5 + 6 = 11 = \frac{1}{H(\frac{1}{X+Y})}$$

Again, if Z is another variable which denotes prime integer lying between 1 & 10 then it assumes the values

1,3,5,7

After calculation, it is found that

$$H(\frac{1}{Z}) = 0..25$$
 i.e. $\frac{1}{H(\frac{1}{Z})} = 4$

The variable X + Y + Z assumes the 100 values

3, 5, 7, 9, 11, 5, 7, 9, 11, 13, 7, 9, 11, 13, 15, 9, 11, 13, 15, 17, 11, 13, 15, 17, 19, 6, 8, 10, 12, 14, 16, 10, 12, 14, 16, 10, 12, 14, 16, 18, 12, 14, 16, 18, 20, 14, 16, 18, 20, 22, 8, 10, 12, 14, 16, 10, 12, 14, 16, 18, 12, 14, 16, 18, 20, 14, 16, 18, 20, 22, 16, 18, 20, 22, 24, 26.

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After calculation, it is found that

Note that

$$\frac{1}{H(\frac{1}{X})} + \frac{1}{H(\frac{1}{Y})} + \frac{1}{H(\frac{1}{Z})} = 5 + 6 + 4 = 15 = \frac{1}{H(\frac{1}{X+Y+Z})}$$

V. DISCUSSION AND CONCLUSION

The additive property of arithmetic mean [2, 5, 21, 26] states that

the arithmetic mean of the sum of a number of variables is equal to the sum of their individual arithmetic means

i.e. if

 X_1 , X_2 ,, X_p

are p discrete random variables such that all assume non-zero values then

$$A(X_1 + X_2 + \dots + X_p) = A(X_1) + A(X_2) + \dots + A(X_p)$$

Thus the additive property of **arithmetic mean** can be summarized as

Arithmetic Mean of Sum of Variables = Sum of Arithmetic Mean of Variables

Theorem (3.1), describing the additive property of harmonic mean, can be summarized as

$$\frac{1}{\text{Harmonic Mean of } \left(\frac{1}{\text{Sum of Variables}}\right)} = \text{Sum of } \left\{\frac{1}{\text{Harmonic Mean of } \left(\frac{1}{\text{Variable}}\right)}\right\} s$$

This is the rhythm lying in the additive property of **harmonic mean**. For this reason, additive property of **harmonic mean** can be regarded as *Rhythmic Additive Property of Harmonic Mean*.

It is to be mentioned that the additive property of **harmonic mean** has here been derived in the case of a discrete random variable. Derivation of this property is to be done in the case of continuous random variable.

It is also required to search for if their exists more property or properties of harmonic mean.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag



(Dr. Dhritikesh Chakrabarty, in the middle, with his students in Statistics Department of Handique Girls' College on his last official working day (December 31, 2021) at the institution)

Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

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Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 270 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS) & (3) Partners Universal International Research Journal (PUIRJ). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.

Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.