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Multiplicative Property of Geometric Mean: Another Proof

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ABSTRACT: Multiplicative property of geometric mean, which had been identified in a study, was derived in that study in the case of discrete variable from its classical definition. Later on, this property of geometric mean was derived in the case of same variable from the additive property of arithmetic mean which is an established one. A third proof of the property has been forwarded in this article.

KEYWORDS*:* Discrete Variable, Geometric Mean, Multiplicative Property, Third Proof

I. INTRODUCTION

Average [4 , 52] is a concept which has been found to be used in almost everywhere. Measure of average was first developed by the great mathematician Pythagoras [7 , 43 , 45 , 48 , 49 , 54]. He defined three measures of average namely arithmetic mean, geometric mean and harmonic mean which were given the name "Pythagorean Means" [6 , 8 , 9 , 10 , 12 , 15 , 35 , 56] as a mark of honour to him. Later on, a number of definitions / formulations of average had been derived due to necessity of handling different situations. Some of them are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized *p mean* & generalized p^{th} root mean etc. in addition to Arithmetic Mean , Geometric Mean & Harmonic Mean [10 , 15 , 29]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [11, 13 , 14 , 17]. Recently, four formulations of average have been derived from the three Pythagorean means which are Arithmetic-Geometric Mean, Arithmetic-Harmonic Mean, Geometric-Harmonic Mean and Arithmetic-Geometric-Harmonic respectively [16 , 19 , 25 , 29]. Pythagorean classical means, along with the other means derived from these three, play vital roles in finding measures of various characteristics of data namely central tendency, dispersion, correlation regression etc. [1 , 2 , 3 , 18 , 19 – 28 , 30 – 34 , 36 , 37 , 42 , 44 , 46 , 47 , 50 , 51 , 53 , 55 , 57]. Various properties of the three Pythagorean means have already been identified which are available in the literature of statistics [6 , 48] while more properties of them are yet to be identified. Recently, one property of harmonic mean which can be termed as its additive property [38 , 40] and one property of geometric mean which can be termed as its multiplicative property [39 , 41] have been identified in the case of discrete variable. The multiplicative property of geometric mean was first derived in the case of discrete variable from its classical definition [39]. It is to be mentioned that the additive property of arithmetic mean is already an established one [38]. Accordingly, the multiplicative property of geometric mean was derived, later on, from the additive property of arithmetic mean [41]. A third proof of the property has been forwarded in this article.

II. AITHMETIC & GEOMETRIC MEANS

Definition

Arithmetic Mean of a list of *N* real numbers or values namely

$$a_1 , a_2 , \ldots\ldots , a_N$$

, denoted by $A(a_1 , a_2 , \ldots\ldots , a_N)$, is defined by

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$$A(a_1 , a_2 , \ldots\ldots , a_N) = \frac{1}{n} \sum_{i=1}^{N} a_i \quad (2.1)$$

while Geometric Mean of them, denoted by $G(a_1 , a_2 , \ldots\ldots , a_N)$, is defined by

$$G(a_1 , a_2 , \ldots\ldots , a_N) = \left(\prod_{i=1}^{N} a_i \right)^{1/N} \quad (2.2)$$

provided the *N* numbers are strictly positive.

Note:

Taking log on both sides of (2.2), it is obtained that

$$\log G(a_1 , a_2 , \ldots\ldots , a_N) = \frac{1}{N} \sum_{i=1}^{N} \log a_i \quad (2.3)$$

Now, if V is a variable which assumes the values

$$v_1 , v_2 , \ldots\ldots\ldots , v_M$$

then Arithmetic Mean of V, denoted by $G(V)$, is defined by

$$A(V) = \frac{1}{M} \sum_{i=1}^{M} v_i \quad (2.4)$$

while Geometric Mean of V, denoted by $G(V)$, is defined by

$$G(V) = \left(\prod_{i=1}^{M} v \right)^{1/M} \quad (2.5)$$

Note:

Taking log on both sides of this it is obtained that

$$\log G(V) = \frac{1}{M} \sum_{i=1}^{M} \log v_i \quad (2.6)$$

Corollary:

(1) If c is a non-zero constant then equations (2.1) & (2.2) yield

$$A(ca_1 , ca_2 , \ldots\ldots , ca_N) = c\, A(a_1 , a_2 , \ldots\ldots , a_N) \quad (2.7)$$
$$\& \quad G(ca_1 , ca_2 , \ldots\ldots , ca_N) = c\, G(a_1 , a_2 , \ldots\ldots , a_N) \quad (2.8)$$

respectively.

In particular, if $c = -1$,

$$A(-a_1 , -a_2 , \ldots\ldots , -a_N) = -A(a_1 , a_2 , \ldots\ldots , a_N) \quad (2.9)$$
$$\& \quad G(-a_1 , -a_2 , \ldots\ldots , -a_N) = -G(a_1 , a_2 , \ldots\ldots , a_N) \quad (2.10)$$

(2) Similarly, if c is a non-zero constant then equations (2.4) & (2.5) yield

$$A(cV) = c\, A(V) \quad \& \quad G(cV) = c\, G(V) \quad (2.11)$$

respectively.

In particular, if $c = -1$,

$$A(-V) = -A(V) \quad \& \quad G(-V) = -G(V) \quad (2.12)$$

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III. MULTIPLICATIVE PROPERTY OF GEOMETRIC MEAN

Statement of the Property:

If

$$X_1 , X_2 , \ldots\ldots\ldots\ldots , X_k$$

are k discrete variables such that all assume non-zero values then

$$G\,(X_1X_2 \ldots\ldots\ldots\ldots X_k) = G\,(X_1)\,G(X_2) \ldots\ldots\ldots\ldots G(X_k)$$

In particular, if X & Y be two variables such that each of them assumes the non-zero values then

$$G\,(XY) = G(X)\,G(Y)$$

Proof:

Let us first consider the case of two variables.
Let X & Y be two variables such that
X assumes the *m* positive values

$$x_1 , x_2 , \ldots\ldots\ldots\ldots , x_m$$

and Y assumes the *n* positive values

$$y_1 , y_2 , \ldots\ldots\ldots\ldots , y_n$$

respectively
such that by equation (2.6),

$$\log G(X) = \frac{1}{m} \sum_{i=1}^{m} \log x_i$$

$$\& \quad \log G(Y) = \frac{1}{n} \sum_{j=1}^{n} \log y_j$$

Then the variable XY assumes the *mn* values

$$x_1y_1 , x_1y_2 , \ldots\ldots\ldots\ldots , x_1y_n ,$$

$$x_2y_1 , x_2y_2 , \ldots\ldots\ldots\ldots , x_2y_n ,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$x_my_1 , x_my_2 , \ldots\ldots\ldots\ldots , x_my_n .$$

By equation (2.6),

$$\log G(XY) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \log (x_i\, y_i)$$

$$= \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (\log x_i + \log y_i)$$

$$= \frac{1}{mn} \left[\sum_{i=1}^{m} \sum_{j=1}^{n} \log x_i + \sum_{i=1}^{m} \log y_{i_i} \right]$$

$$= \frac{1}{mn} \left[n \sum_{i=1}^{m} \log x_i + m \sum_{j=1}^{n} \log y_{i_i} \right]$$

$$= \frac{1}{m} \sum_{i=1}^{m} \log x_i + \frac{1}{n} \sum_{j=1}^{n} \log y_{i_i}$$

$\Rightarrow \log G(XY) = \log G(X) + \log G(Y)$, by equation (2.6)

$\Rightarrow \log G(XY) = \log \{G(X)\, G(Y)\}$

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Hence, $G(XY) = G(X)\,G(Y)$ (3.1)

Now, if X , Y & Z are three variables each of which assumes positive values then

$$\begin{aligned} & G(XYZ) = G\{(XY)Z\} \\ \Rightarrow\ & G(XYZ) = G(XY)\,G(Z) \text{ , by equation (3.1)} \\ \Rightarrow\ & G(XYZ) = G(X)\,G(Y)\,G(Z) \text{ , by equation (3.1)} \end{aligned} \quad (3.2)$$

By the same logic, it is obtained for four positive valued variables X , Y , Z & W that

$$G(XYZW) = G(X)\,G(Y)\,G(Z)\,G(W) \quad (3.3)$$

Applying the same logic to the variables

$$X_1 , X_2 , \ldots\ldots\ldots\ldots , X_k ,$$

it can be obtained that

$$\boldsymbol{G}(X_1X_2 \ldots\ldots\ldots\ldots X_k) = \boldsymbol{G}(X_1)\,\boldsymbol{G}(X_2) \ldots\ldots\ldots\ldots \boldsymbol{G}(X_k) \quad (3.4)$$

Note:

If

$$c_1 , c_2 , \ldots\ldots\ldots\ldots , c_k ,$$

are non-zero constants,

then applying equation (3.4) to the variables

$$c_1X_1 , c_2X_2 , \ldots\ldots\ldots\ldots , c_kX_k ,$$

it is obtained that

$$G\{(c_1X_1).(c_2X_2).\ldots\ldots\ldots.(c_kX_k)\} = (c_1.c_2).\ldots\ldots\ldots.c_k)\{G(X_1).G(X_2).\ldots\ldots\ldots.G(X_k)\} \quad (3.5)$$

This can be termed as general multiplicative property of geometric mean.

Corollary:

Putting the values of all

$$c_1 , c_2 , \ldots\ldots\ldots\ldots , c_k ,$$

as – 1 in equation (3.5),

it is obtained that

$$G\{(-X_1).(-X_2).\ldots\ldots\ldots.(-X_k)\} = (-1)^k\{G(X_1).G(X_2).\ldots\ldots\ldots.G(X_k)\} \quad (3.6)$$

IV. DISCUSSION AND CONCLUSION

The multiplicative property of geometric mean was derived from its classical definition the in earlier study [39]. Later on, this property has been derived from the additive property of arithmetic mean [41]. A third proof of the property has been forwarded in this article. The aim of this study was to verify whether the three tracks of derivation yield the same result and in the study it has been found so. Consequently, the correctness of the multiplicative property of geometric mean, as obtained in the earlier study, has also been established by this study.

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In this connection, it is to be mentioned that development of the theory of geometric mean still lies in the immature stage. Lot of studies are still pending on various aspects of geometric mean specifically on its other possible properties, its applications and many others.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

(Dr. Dhritikesh Chakrabarty, standing in middle, with his teacher Sjt. Satyendra Nath Sharma, second from the right in front, and some other academicians in the Golden Jubilee Celebration of Department of Statistics of Darrang College, Tezpur, Assam, held during 06 - 07 February, 2016)

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then

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Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 260 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS) & (3) Partners Universal International Research Journal (PUIRJ). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.

Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.