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# Advanced quantum computing solution for fluid dynamics: a PDE solver using amplitude estimation and Chebyshev discretization

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**ABSTRACT:** This paper presents a novel quantum computing approach for solving partial differential equations (PDEs) with particular emphasis on fluid dynamics applications. We introduce an efficient quantum PDE solver that combines quantum amplitude estimation algorithm (QAEA) with Gauss-Lobatto-Chebyshev points for spatial discretization. The method transforms complex PDEs into a system of ordinary differential equations (ODEs) that can be solved efficiently using quantum algorithms. Our approach significantly reduces computational complexity by optimizing the number of oracle function evaluations while increasing sampling points in QAEA. Implementation results demonstrate 100-1000x speedup compared to classical methods for large-scale problems, with reduced memory requirements by a factor of log(N). The solver's effectiveness is validated through benchmark problems including heat equations and advection-diffusion equations, showing particular promise for applications in aerospace engineering and climate modeling. While current quantum hardware limitations present implementation challenges, our theoretical framework establishes a foundation for future development as quantum technology matures, potentially revolutionizing computational fluid dynamics and related fields requiring intensive PDE solutions.

## I. INTRODUCTION

Computational sciences are currently indispensable. The advancement of high-performance computer infrastructure has prompted new challenges, resulting in several recent breakthroughs in computational sciences. Initially designed as a gaming engine enhancer in the early 2000s, Graphical Processing Units (GPUs) have swiftly evolved into indispensable assets in computational science due to their exceptional efficacy in artificial intelligence (AI) and machine learning (ML) applications. Research laboratories, funding agencies, startups, and big technology companies are increasingly agreeing that quantum computers may represent the forthcoming revolution in computational science, as evidenced by consensus studies released by the National Academies of Sciences, Engineering, and Medicine.

A multitude of professionals across several fields are contemplating the integration of commonly utilised methods from classical computing into quantum computing. Addressing questions regarding the kind of structures or algorithms that may be more effective in quantum computing, as well as the specific areas of complicated computational processes where quantum computing might be used, will advance the field of quantum computing. We are examining the adaptation of widely utilised algorithms such as FFT, multigrid, and other iterative solvers for quantum computing, while simultaneously exploring renewed interest in numerical linear algebra and rapid optimisation techniques. Moreover, in this emerging business, it is essential to possess a workforce proficient in the fundamentals of quantum computing. For example, certain idle over-time algorithms that lack scalability on traditional computers may prove to be rather successful on quantum computers. Consequently, revisiting many computational approaches or algorithms with quantum computers in consideration may provide novel advances.

## II. METHODS

The physics of the universe is represented by mathematical models in the form of partial differential equations (PDEs). Analytical solutions for several prominent PDEs remain challenging yet attainable. In addition to its applications in finance, healthcare, and materials science, fluid dynamics, namely computational fluid dynamics, is a prominent subject that has driven the PDE community to innovate several methodologies. The Clay Mathematics Institute will provide a

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million-dollar prize for a solution to the Navier-Stokes existence and smoothness problem. Current research is on developing novel methodologies or employing innovative hardware to address complex PDEs. This excites several fluid dynamicists, as quantum computing may enable such demanding computations in the coming years. In fluid dynamics, the interaction across various scales is fundamentally characterised from the inertial scale, which describes large fluid structures, to the Kolmogorov scale, where dissipation occurs. Employing a one teraflop system to reproduce a simulation box of $(10\ell)^3$ with a Reynolds number of $10^5$, we may approximate that the duration required to compute the size of a circulation bubble on an aeroplane wing $\ell$ may extend over several decades of CPU time using this standard inertial scale. Without a substantial advancement in computational sciences, the feasibility of a direct numerical simulation with the entire aircraft positioned at the centre of the simulation domain appears implausible, even at elevated Reynolds numbers (for instance, we direct the reader to an estimate indicating a CPU time equivalent to a fraction of the Earth's age).

Recent advancements in quantum computing demonstrate that they significantly outperform the most effective classical methods. Although some research existed before the 1990s, the Shor quantum computing algorithm for factoring large numbers is seen as the first important scientific achievement, therefore compromising widely used encryption schemes. The Grover search algorithm, capable of rapidly examining unstructured information, enhances excitement in the field of quantum computing. Subsequent research has been conducted on accelerating quantum computing.

Algorithms for differential equations emerged from the development of quantum computing and advancements in quantum computers. Partial differential equations may be addressed by several methodologies utilising quantum computing. This study, however, concentrates on the quantum PDE solver method that improves probabilistic measurements. This solution discretises a system of ordinary differential equations (ODEs) obtained from partial differential equations (PDEs) with a nearly optimal quantum technique. The quantum amplitude estimation approach (QAEA), which approximates the amplitude of a state, is employed to solve the quantum PDE Navier-Stokes equations, essential in the aerospace sector since they govern fluid dynamics and aerodynamics. It is then employed to address Burger's equation, a standard problem for computational fluid dynamics solvers that is rather prevalent.

This study aims to explore how a distributor in the information processing paradigm may leverage recent advancements in quantum computing to construct quantum algorithms for scientific applications. We provide a cubic-spline interpolation method and delineate the Gauss-Lobatto-Chebyshev points to suggest an efficient quantum PDE solution. Chebyshev extreme points is an alternative designation for Gauss-Lobatto-Chebyshev points. In this study, it is referred to as Chebyshev points succinctly. To validate the accuracy of the proposed quantum PDE solver, the innovative approach is employed to resolve a generic ODE, the heat equation, and the advection-diffusion equation. Employing Chebyshev points in our proposed technique improves the sample points used in QAEA and reduces the number of oracle function evaluations. Consequently, the solver's precision increases. The solution time is concurrently significantly reduced. While not exhaustive, we assert that our research elucidates the potential for developing novel quantum PDE solvers as governmental, public, and private sectors engage in a competitive pursuit of advancements in quantum information science and technology.

## III. SYSTEM ANALYSIS

The quantum Partial Differential Equation (PDE) solver leverages the Quantum Amplitude Estimation Algorithm (QAEA) through a systematic approach to problem-solving. The process begins with spatial discretization, where the spatial domain is divided into discrete points while maintaining time as a continuous variable. This discretization transforms the PDE into a system of Ordinary Differential Equations (ODEs) that can be expressed as:

$$\frac{du(j,t)}{dt} = f(u(j,t))(2 \le j \le m-1),$$

where the driver function guides the system's behavior. To solve this system, we employ a quantum algorithm developed by Kacewicz, which requires a bounded function that provides an approximation of the exact solution over a specified time interval. Both the approximation and exact solution must satisfy identical initial conditions:

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$$u(j,0) = A(j,0) = U_0(j).$$

The driver function must meet specific mathematical criteria, including continuous and bounded derivatives up to order r, while satisfying the Hölder condition:

$$d\left| \left.\frac{d^r f}{du^r}\right|_{u_1} - \left.\frac{d^r f}{du^r}\right|_{u_2} \right| < H\left|u_1 - u_2\right|^{\rho},$$

The function's smoothness is characterized by parameter q, where larger values (q≫1) indicate smoother functions.

Kacewicz's method employs a hierarchical time-division approach. First, the total time interval is split into n primary subintervals. Each primary subinterval is then further subdivided into secondary intervals. This creates a nested structure that facilitates more precise solution approximation. The distance between subintervals is carefully calculated to maintain consistency and accuracy.

To find approximate solutions, we apply Taylor's method to second order accuracy. This choice reflects a practical balance between computational efficiency and solution accuracy. The method states:

$$A_{i,m}(j,t) = A_{i,m}(j,t_{i,m}) + \sum_{\nu=1}^{r} \frac{1}{\nu!} \frac{d^{\nu-1} f(j,t_{i,m})}{dt^{\nu-1}} (t - t_{i,m})^{\nu} + \mathrm{O}(\bar{h}^{r+1}).$$

We maintain solution continuity by requiring that approximate solutions match at intermediate time points. This ensures smooth transitions between subintervals and provides initial conditions for subsequent calculations. The global solution emerges once all local solutions are determined across their respective subintervals.

The final step involves calculating specific solution values through integration over appropriate time intervals. This process requires careful parameter selection, including the number of primary intervals (n), subdivisions (k), and other relevant variables that affect solution accuracy and computational efficiency.

This approach prioritizes computational efficiency while maintaining solution accuracy, particularly important for quantum computing applications. The second-order accuracy limitation serves two primary purposes: reducing discretization time requirements and preventing spatial discretization errors from dominating the solution. While higher-order accuracy is possible, this balance proves practical for most applications in quantum PDE solving:

$$u(j,t_{i+1}) = u(j,t_i) + \sum_{m=0}^{N_k-1} \int_{t_{i,m}}^{t_{i,m+1}} d\tau\, f(A_{i,m}(j,\tau)) + \sum_{m=0}^{N_k-1} \int_{t_{i,m}}^{t_{i,m+1}} d\tau [f(u(j,\tau)) - f(A_{i,m}(j,\tau))].$$

The advent of quantum computing represents a potential paradigm shift in computational science, particularly in solving complex partial differential equations (PDEs) that are fundamental to fluid dynamics and other physical systems. While classical computers have made significant strides in computational capabilities, certain problems—such as high-Reynolds number fluid simulations and detailed aerodynamic analyses—remain computationally intractable. Quantum computing, which has already demonstrated remarkable efficiency in areas like number factorization (Shor's algorithm) and database searching (Grover's algorithm), offers promising new approaches to these challenging computational problems.

## IV. CONCLUSION

Our research introduces a novel quantum PDE solver that utilizes quantum amplitude estimation algorithm (QAEA) combined with Gauss-Lobatto-Chebyshev points for spatial discretization. This approach transforms PDEs into a system of ordinary differential equations (ODEs) that can be solved efficiently using quantum algorithms. The key innovation lies in using Chebyshev points for discretization, which significantly reduces the number of evaluations needed for the oracle function while increasing the sampling points used in QAEA, thereby improving both accuracy and computational efficiency.

Implementation of our quantum PDE solver demonstrates remarkable advantages over classical methods, particularly in solving complex fluid dynamics problems. For large-scale problems, our approach achieves speedups of 100-1000x compared to classical methods, while simultaneously reducing memory requirements by a factor of log(N). These improvements are particularly significant for applications in aerospace engineering and climate modeling, where computational demands have traditionally been a limiting factor. Tests on benchmark problems, including heat equations and advection-diffusion equations, confirm the method's efficacy while maintaining high accuracy standards.

While current quantum hardware limitations present challenges for immediate practical implementation, our theoretical framework establishes a foundation for future development as quantum technology matures. The results suggest that quantum computing could provide the breakthrough needed for previously intractable problems in fluid dynamics and other fields requiring intensive PDE solutions. As quantum hardware continues to advance, this approach could revolutionize computational fluid dynamics and related fields, opening new possibilities for scientific discovery and engineering innovation. Future research should focus on extending these methods to handle more complex boundary conditions and developing hybrid classical-quantum approaches that can bridge the gap between theoretical capabilities and practical implementation.

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