Graphoidal Cover Independence Number of a Graph

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ABSTRACT: Let G be a graph. Let $\psi$ be any acyclic graphoidal cover of G. The minimum cardinality of the maximum $\psi$-independence set is called graphoidal cover independence number of G and is denoted by $\beta_{\psi_\circ}$.

In this paper, we find graphoidal cover independence number for some standard graphs and some interesting results.

I. INTRODUCTION

The concept of graphoidal cover and graphoidal covering number of a graph G was introduced by Acharya and Sampathkumar [2].

A graphoidal cover of a graph G is a collection of $\psi$ of paths(not necessarily open) in G satisfying the following conditions.
(i) Each path in $\psi$ has at least two vertices.
(ii) Every vertex of G is an internal vertex of at most one path in $\psi$.
(iii) Every edge of G is in exactly one path in $\psi$.

Arumugam and Suresh Suseela [4] introduced the concept of acyclic graphoidal cover and acyclic graphoidal covering number of a graph G.

A graphoidal cover $\psi$ of a graph G is called an acyclic graphoidal cover if every member of $\psi$ is a path. Let $\psi$ be any acyclic graphoidal cover of G. The maximum cardinality of maximal $\psi$-independence set is called $\psi$-independence number of a graph G and is denoted by $\beta_{\psi_\circ}(G)$.

II. MAIN RESULTS

Example 1.1.

Consider the graph G and the corresponding $\psi$-graph. Let $\psi = \{\{u_1, u_2\}\} \cup E(G)$. Then $\beta_{\psi_\circ}(G) = n - 1$. 

[Diagram of graph G and graph G(\psi).]
Example 1.2.
Consider the graph $G$ and the corresponding $\psi$-graph. Let $\psi = \{(v_1, u_1, u_2, \ldots , u_n)\} \cup E(G)$. Then $\beta (G) = n + 1$. So that $H = G$. The graph obtained from the cycle $C = (v_1, v_2, \ldots , v_n)$ by attaching three pendant edges to every vertex of $C$. Let $x_i, y_i$, and $z_i$ (for $i \leq n$) be the pendant vertices which are adjacent to $v_i$.

Then, $\psi = \{x_i, x_j, y_i, z_i, z_j : 1 \leq i, j \leq n\} \cup E(G)$ is an acyclic graphoidal cover of $G$. Further $G(\psi)$ is isomorphic to $C \circ K_n \cup K_2$ so that $\beta (G) = 2n$. Also $\beta_0 (G) = 3n$ and we have $\beta (G) - \beta_0 (G) = n$.

Theorem 1.4. Given any positive integer $n$, there exists a graph $G$ and an acyclic graphoidal cover $\psi$ of $G$ such that $\beta (G) - \beta_0 (G) = n$.

Proof. We construct a graph $G$ as follows. Let $G$ be the graph obtained from the cycle $C = (v_1, v_2, \ldots , v_n)$ by attaching three pendant edges to every vertex of $C$. Let $x_i, y_i$, and $z_i$ (for $i \leq n$) be the pendant vertices which are adjacent to $v_i$.

Then, $\psi = \{x_i, x_j, y_i, z_i, z_j : 1 \leq i, j \leq n\} \cup E(G)$ is an acyclic graphoidal cover of $G$. Further $G(\psi)$ is isomorphic to $C \circ K_n \cup K_2$ so that $\beta (G) = 2n$. Also $\beta_0 (G) = 3n$ and we have $\beta (G) - \beta_0 (G) = n$.

Theorem 1.5. Given any positive integer $n$, there exists a graph $G$ and an acyclic graphoidal cover $\psi$ of $G$ such that $\beta (G) - \beta_0 (G) = n$.

Proof. We construct a graph $G$ as follows. Let $H = K_{1,n+1}$, Let $V(H) = \{v, v_1, \ldots , v_n\}$ be the vertex set of $H$ and $v$ be the centre vertex of $H$. Subdivide $H$ by once and $u_1, u_2, u_3, \ldots , u_n$ be the subdividing vertices and attach a pendant to the vertex $v$ and call it as $w$ and denote the graph as $G$. Now $|G| = 2n + 2$. Let $\psi$ be any acyclic graphoidal cover of $G$. Let $S$ be the set of vertices which are interior to $\psi$. Let $S = \{u_1, u_2, u_3, \ldots , u_n\}$ and $\psi = \{vu_1, v_1, v_2, \ldots , v_n\}$. Then $G(\psi)$ is isomorphic to $K_{1,n+1} \cup (nK_1)$ and $\beta (G) = n + 1$. Also $\beta_0 (G) = n + 1$, so that $\beta (G) - \beta_0 (G) = 2n + 1 - (n + 1) = n$.

Problem 1.6. Does there exist a graph $G$ such that

(i) $\beta (G) - \beta_0 (G) > n$

(ii) $\beta (G) - \beta_0 (G) < n$
Remark 1.7. For any two minimum graphoidal covers $\psi_1$ and $\psi_2$, $\beta_{0\psi_1} \neq \beta_{0\psi_2}$ and not isomorphic also. For example, consider the graph $G,$

Let $\psi_1 = \{(1 3 2)(5 7 6)(9 8 10)(13 11 14)(7 4 3)(8 4)(4 11)(11 12)\}$, then $G(\psi_1)$ is given below.
Let $\psi_2 = \{(1\ 3\ 2)(5\ 7\ 6)(9\ 8\ 10)(13\ 11\ 14)(8\ 4\ 11)(11\ 12)(7\ 4)(4\ 3)\}$, then $G(\psi_2)$ is given below.

![Graph](image)

**Theorem 1.8.** Let $T$ be a tree. Then for any minimum acyclic graphoidal cover $\psi$ of $T$, $\beta_0(\psi) = n-1$, if and only if $T = P_n$, where $n$ is the number of vertices.

**Proof.** Let $T = P_n$, then obviously $\beta_0(\psi) = n-1$. If $\beta_0(\psi) = n-1$, then we have to prove that $T = P_n$. Suppose $T \neq P_n$. Then at least one vertex of degree $\geq 3$. Then all vertices of degree $\geq 2$ are interior to $\psi$, since $\psi$ is a minimum graphoidal cover and $G(\psi)$ is isomorphic to $(n - 4)K_1 \cup 2K_2$ and $\beta_0(\psi) = n - 2$ which is a contradiction. This completes the proof.

**Problem 1.9.** Characterise the graphs $G$ for which $\beta_0(\psi) = n-1$.

**Theorem 1.10.** For any acyclic graphoidal cover $\psi$ of the path $G = P_n$ on $n$ vertices,

$$\beta_0(\psi) = n - (k + 1) + \left\lfloor \frac{k + 1}{2} \right\rfloor$$

where $k = |\psi|$.

**Proof.** Let $G = P_n$ be a path on $n$ vertices. It is clear that for any graphoidal cover $\psi$ of $G$, $G(\psi)$ is isomorphic to $P_{k+1} \cup (n-(k+1))K_1$ where $k = |\psi|$, so that $\beta_0(\psi) = n - (k + 1) + \left\lfloor \frac{k + 1}{2} \right\rfloor$.

**Theorem 1.11.** For any acyclic graphoidal cover $\psi$ of the cycle $G = C_n$ on $n$ vertices, $\beta_0(\psi) = n - k + \left\lfloor \frac{k}{2} \right\rfloor$

where $k = |\psi|$.
Proof. Let $G = C_n$ be a cycle on $n$ vertices. It is clear that for any graphoidal cover $\psi$ of $G$, $\psi(G)$ is isomorphic to $C_k \cup (n-k)K_1$ where $k = |\psi|$, so that $\beta \psi(G) = n - k + \left\lceil \frac{k}{2} \right\rceil$.

**Theorem 1.12.** Let $T$ be any tree with $n$ pendant vertices and let ‘r’ denote the number of vertices of degree two. Then for any minimum acyclic graphoidal cover $\psi$, $\beta \psi(T) \leq n + r - 1$.

**Proof.** Let $\psi = \{P_1, P_2, P_3 ... P_{n+1}\}$ be a minimum acyclic graphoidal cover of $T$. Let $v_i, 1 \leq i \leq n-1$ be an end vertex of $P_i$. Then $D = \{v_1, v_2, v_3 ... v_{n-1}\} \cup \{u_1, u_2, u_3 ... u_r\}$ where $u_1, u_2, ... u_r$ are the vertices of degree two is an independence of $T(\psi)$. Hence $\beta \psi(T) \leq n - 1 + r = n + r - 1$.

**Problem 1.13.** Characterize the graphs $G$ for which $\beta \psi(G) = n + r - 1$.

**III. CONCLUSION**

We find graphoidal cover independence number for some standard graphs and some interesting results and also we can find graphoidal cover independence number for any graph.

**REFERENCES**


